STRIPLINE FED LOG PERIODIC ARRAY OF SLOTS

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AUGUST, 1989

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by
ARINDAM CHAKRABARTI

to the
DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
AUGUST, 1989

CERTIFICATE

This is to certify that the work presented in this thesis titled, " Stripline fed log periodic array of slots " has been carriedout by Arindam Chakrabarti under my supervision and the same has not been submitted elsewhere for a degree.

Dated : 14 Aug 1989

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ACKNOWLEDGEMENT

I express a strong gratitude to Dr. M. Sachidananda whose valueable suggestions and comments have greatly improved the thesis.

I am grateful too, to my friends who have helped me in innumerous ways for the present - work.

To those for whom this work will be
of some use.

ABSTRACT

A general formulation for the stripline fed log periodic slot array on the upper ground plane of a stripline is presented. A single slot on the upper ground plane of a stripline is analysed and an equivalent slot impedance is derived for the purpose of array design. In the single slot analysis, the variation of normalized resonant resistance with strip offset and the variation of slot impedance with frequency are studied. It was found that the resonant slot length is less than half wavelength. An array of such slots of unequal lengths placed symmetrically with respect to the center of the stripline is analysed, taking external mutual coupling among slots into account. An expression for effective impedance of a single slot in the presence of other slots is derived and an iterative procedure to evaluate the effective slotimpedanceis described. Then, a general discussion on a log periodic slot array, where the lengths increase in a geometric progression from the feed point, is presented. Various quantities and parameters involved in the design of such an array are defined. Two methods of attack for such an array design are discussed and are named as symmetric and asymmetric design procedures. Both the methods are applied to two such arrays or units placed back to back on the same upper ground plane of the stripline. Each unit is terminated by a matched load at the low frequency end and are fed from a 3 dB 0° - 180° hybrid at the high frequency ends. The maximum VSWR over an octave band of frequency (2-4 GHz) with variation of various parameters are plotted. A procedure to select parameters of this array with an input match over an operating band width is described and in both the design procedures these parameters are selected. It was observed that asymmetric design offers smaller area of the substrate material. It was also observed that beyond the operating bandwidth, although the input VSWR is low, the power radiated by the antenna is small and most of the power is absorbed by the matched termination. Extension of present work for future development of wideband antennas and slot arrays is discussed.

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CHAPTER I

INTRODUCTION

1.1. INTRODUCTION :

The search for wideband antennas whose input characteristics and pattern remain same with frequency is very old. Many antennas such as, long wire antenna, biconical antenna notch antenna etc., are broad band antennas but their patterns do not remain same with frequency. On the otherhand, the spiral antenna and the log periodic antenna are broad band as well as their pattern remains the same with change of frequency.

While designing an antenna for given electrical specifications of input bandwidth, pattern shape, gain etc., it is also desirable to keep the cost low. This consideration has led to the investigation into the type of antennas that can be fabricated using printed circuit technology especially at higher microwave and millimeter wave frequencies. There are a large variety of antennas such as, microstrip patch, printed dipole, spiral, stripline or microstrip fed slot array etc., which have been successfully designed and fabricated. The present work reported in this thesis adds one more to this long list of antennas which are fabricated using printed circuit technology. The special feature of the antenna considered is the constancy of pattern over a broad frequencyband.

What was saught to be achieved in this study is to design an antenna with a pattern similar to the broadside pattern of a two element array of dipoles kept $\lambda/2$ apart and vertical, but with horizontal polarization and one octave bandwidth. An additional constraint imposed is that the pattern be same over the entire operating frequency band.

The obvious choice, from the polarization point of view, was an array of slots placed $\lambda/2$ apart in a ground plane. Of course, this gives dipole like pattern only in one half space, if otherside of the slot is used for excitation, but that is the type of pattern required in most applications.

Now, the behaviour of the slots is like resonant dipoles with small bandwidth. Therefore, we need to devise a mechanism of obtaining broad bandwidth. The additional constraint of making the pattern also constant with respect to frequency requires that the slot spacing also be same in terms of λ,

the wavelength.

The configuration arrived at based on these criteria is a pair of log periodic slot array fed in some convenient way. The feeding method selected is using a strip transmission line, which is easy to fabricate using the printed circuit technology used for slot fabrication.

Here, the concept of log periodic dipole array (LPDA) has been borrowed and applied to a slot array, but with some modifications. The conventional LPDA has an endfire pattern but in the present study, the feed structure is designed for a broadside pattern.

The LPDA is a wellknown type of antenna which has been in use for a long time. The analysis and design of such an array has been adequately studied and is available in standard text-books. There have been studies of LPDA fabricated using printed dipoles especially in microwaveregion [7].

The present configuration is somewhat different than these, in the sense that the dipoles have been replaced by slots and the feeding of slots is designed for braodside pattern. The study of individual slots fed by stripline has been studied by B.N. Das and K.V. Prasad [1]. Thus, basic studies needed for design of present configuration already exists. These analysis procedures and the concept used in LPDA are made use of in designing Log Periodic Array of Stripline fed slots (LPASS).

Two such LPASS arrays are combined to produce a two slot array. pattern. At any given frequency only one or two slots in each of the arrays resonate or radiate and the spacings between the LPASS are adjusted to maintain the spacing between the radiating slots same in terms of the wavelength.

The word log periodic is explained in detail in chapter 5, and it is sufficient here, if we say that the Log Periodic Array means, the lengths, widths and the spacings between slots increase or decrease in a Geometric Progression (G.P.) as they (slots) are traversed from one end to the other. In the logarithmic scale, these become periodic. In fig. 5.1 in chapter 5, a single unit LPASS is shown. An LPASS antenna produces a beam in broadside direction provided the phases are chosen appropriately at each slot for excitation. If the slots, in any unit of LPASS are resonant at f_1, f_2, f_3, ..., f_n frequencies, where these frequencies form a G.P. sequence; at any such f_i a single slot will be resonant and the pattern will be on the broadside and as the frequency is

increased, resonance shifts and almost the same input impedance and pattern will be observed. Two such LPASS units are joined back to back separated by a distance Ds (see fig. 5.2), and the slots being symmetrically placed about the common axis, i.e., about the middle strip of the stripline. By the same principle of operation, as explained earlier, in this case, two slots, one from each LPASS unit, will be resonant and the distance between the resonant slots of each unit will be same in terms of λ at all the resonant frequencies. Since in our case, a single beam is prefarred, Ds may be chosen as $\lambda_1/2$ where λ_1 is the resonant wavelength of the first slot in the LPASS as shown in fig. 5.1. Also, for a single beam on the broadside, these two LPASS units are fed from a 3dB hybrid power divider with their phases differing by 180° [see fig.1.1]. In an LPDA or LPASS, if the number of dipoles or slots is infinite, starting from an infinitely small dipole with the spacing between two dipoles growing from infinitely small to infinitely large, and with infinitely small common ratio of the G.P. sequence of their growth, the bandwidth will be infinite. But in practical situation the number of dipoles is finite, hence the bandwidth is limited for such an LPDA or LPASS. In our case, a specific bandwidth (2-4 GHz) is chosen and the input characteristics, VSWR etc. are studied within this band as well as a little beyond it on both sides.

To arrive at the design equations, a single slot on the upper ground plane of a stripline is discussed and the impedance offered by it over a range of frequency is calculated. Then the expression for mutual impedance offered by two such slots of unequal lengths placed side by side on the same upper ground plane is derived. Finally we entered into the problem of an array of unequal slots and the effective impedance at a single slot is found out. Also the input impedance of any such LPASS unit is derived. With this analytical background, a programme is written to calculate the maximum VSWR over (2-4GHz) bandwidth, with the change of various parameters like common ratio of the G.P., distance between slots etc.. The input VSWR over the frequency band 2-4 GHz is evaluated for various parameters, to study the effect of each. From these characteristics the best design values for array dimensions are chosen. The maximum VSWR allowed at the input is chosen to be 1.9 over the 2-4 GHz bandwidth in determining the design values of the array.

1.2. SURVEY OF THE LITERATURE :

We have seen the essential feature of LPASS i.e., as the frequency is changed from low to a high value the resonance shifts from one slot to another and the radiation pattern and input impedance of the antenna system remain the

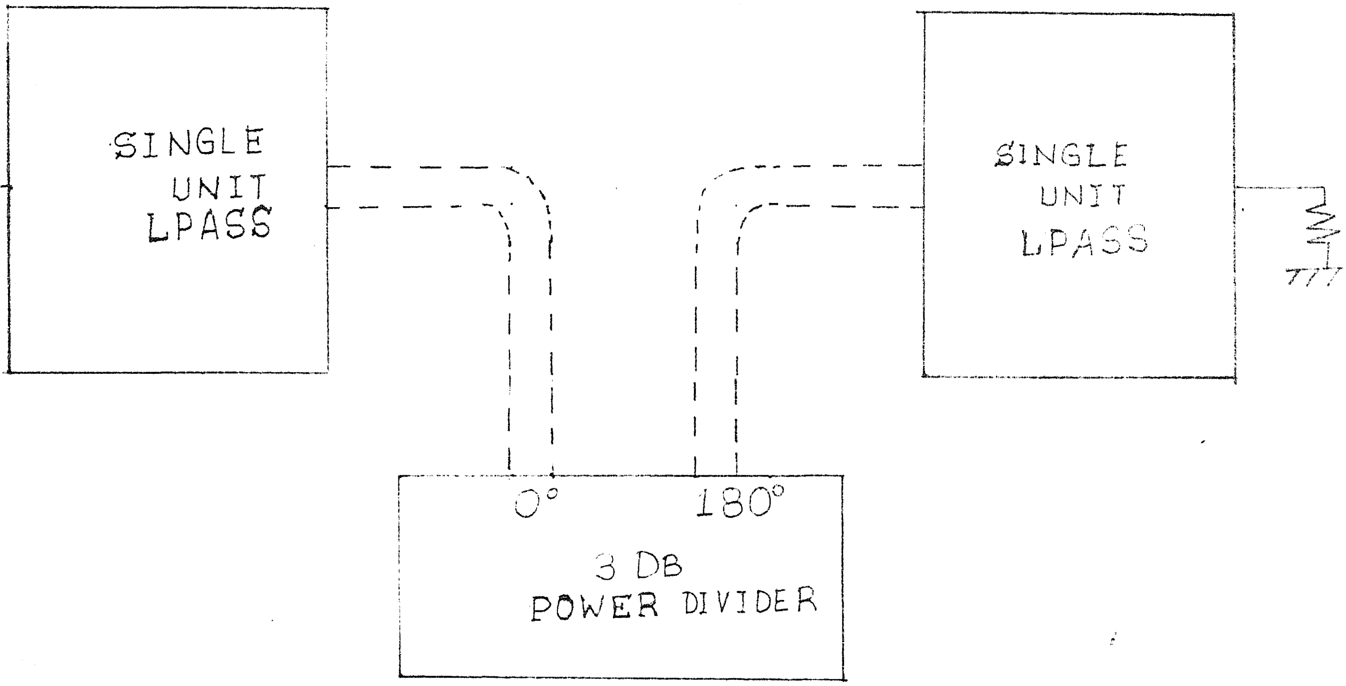


Fig 1.1. Two LPASS units fed by a 3 - dB, 0° - 180° Power Divider. Each unit is terminated by a matched load.

same. So, if the radiating surface of an antenna remains same with frequency it acts as a broad band antenna. Elliott [5] has discussed this aspect of spiral antenna which is also the basis of Log Periodic Antenna. Several authors have contributed to the design and analysis of LPDA. King's approach [6] was that of using three term theory for current distribution and study of LPDA over a large bandwidth.

As for the slot array Elliott and Shavit [2] have given the generalized formulation of transverse slot array on a boxed stripline. They took dipoles of equal lengths which are $\lambda/2$ apart and placed asymmetrically with respect to the middle strip of the stripline. B.N. Das and Prasad [1] have formulated the expression for impedance of a single slot on the upper or lower ground plane of a stripline. They calculated the discontinuity in modal voltage (ΔV) and found impedance by dividing the square of ΔV by the complex power radiated by the slot.

These two ideas will be fused together, for the formulation of a part of our present problem. Elliott [3] has also presented an analysis for mutual impedance due to external coupling between slots appearing in an array, which is further discussed later. This idea is utilized to calculate the mutual coupling between slots.

1.3. ORGANIZATION OF THE THESIS :

Chapter 2 discusses in brief, the single slot theory and is extended further to serve the purpose of analysis in the present problem. The impedance due to a single slot and an equivalent network for the slot is also derived.

In chapter 3, the expression for the mutual coupling between two slots are derived using the method presented by R.S. Elliott [3]. Receiprocity principle is applied directly and the coupling voltage due to the mutual interaction between slots is determined.

In chapter 4, the general problem of stripline-fed slot array is discussed. The expression for active slot impedance of a given slot in the presence of other slots, is derived and along with it the equivalent input impedance is also determined.

The analysis and design equations of log periodic array of slots is taken up in chapter 5. The LPASS characteristics with variation of design parameters

are discussed, which are used to obtain optimum design parameters of LPASS antenna.

Finally in chapter 6, a specific design of LPASS for operating frequency range of 2-4 GHz is presented. Some discussions on the scope for further investigations in the same area are given in the concluding remarks.

CHAPTER - 2

ANALYSIS OF STIPLINE-FED SLOT ANTENNA

2.1 INTRODUCTION AND METHOD OF ANALYSIS :

The basic module for a transverse slot-fed by a boxed stripline is shown in fig. 2.1. The walls of the box are assumed to be perfect conductors. The middle stip is at a distance h_1 from the bottom ground plane and of width w. The waveguide is filled with a homogeneous, isotropic dielectric of relative dielectric constant εr. Pin curtains are inserted at each end of the module.

In response to the passage of an incident transverse electromagnetic (TEM) wave of amplitude A_T, an electric field develops in the slot. Since the slot is very narrow ($d<<\lambda$) the electric field is symmetric about the z axis and sinusoidal in nature.

A direct consequence of this symmetry is that the back scattered wave amplitude B_T and forward scattered wave amplitude C_T are equal in magnitude but of opposite sign. This is equivalent to the scattering from a series lumped impedance Z in a transmission line of characteristic impedance Zo.

In the absence of pin curtains (which are placed very close to the slot), the scattered wave consists of TEM as well as all higher order modes. The pin curtains suppress all higher order modes except TEM. The slot voltage now will be a superposition of all these modes. And the discontinuity in modal voltage (ΔV) will also contain all these terms. It turns out that TE_{10} (Transverse Electric) mode is a propagating mode as per our guide dimensions but all higher order modes are evanescent modes. The presence of pin curtains form a cavity for all these modes except TEM mode as this mode passes through the pin curtains. Since the longitudinal distance between the pin curtains is very small (around 1 - 2mm.) compared to the wavelength of operation, the energy stored in these modes is negligible. The propagating mode TE_{10} gets arrested by the pin curtains. Hence in analysis, only the TEM mode scattering is considered. Now, the discontinuity in modal voltage is due to the back and forward scattered amplitudes B_T, C_T only. The complex

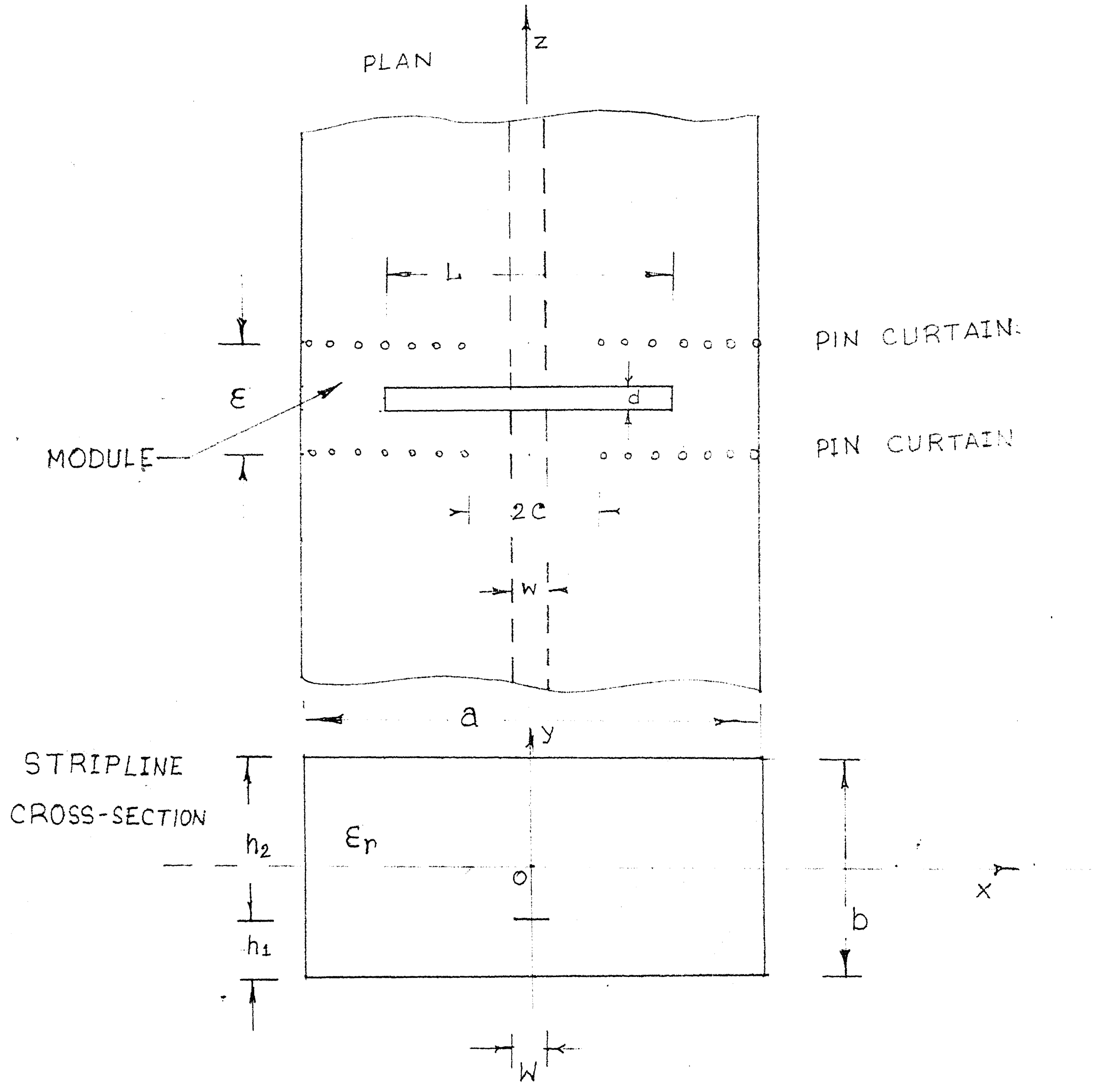


Fig 2.1. The basic slot module and the cross-section of an offset stripline.

power (P) radiated by the slot is evaluated [1] and the ΔV^2 divided by this power gives the equivalent series impedance seen by the stripline. This analysis follows the procedure given by Das and Prasad [1].

2.2 ANALYSIS :

The geometry of the transverse slot on the upper ground plane of an offset stripline is shown in fig. 2.2(a). Also the equivalent circuit of the transverse slot is shown in fig. 2.2(b). Let a TEM wave of amplitude A_T is incident on the slot from -z direction. Since the slot interrupts the surface current, it develops an electric field $\overline{E^S}$ across it. This slot field in turn scatters the incoming wave resulting in reflection of the wave of amplitude B_T and transmission of wave of amplitude $(A_T + C_T)$, where B_T and C_T are the back and forward scatter co-efficients. It is assumed that the line is terminated in a matched load at $z > z_n + d/2$.

We can view the situation in the following way : there is a magnetic current source at $z=z_n$ and that this source produces two scattered waves with amplitudes B_T and C_T propagating in -z and +z directions respectively. Both sides of $z=z_n$ are assumed to be matched. The slot electric field is shown in fig. 2.4 The scattered electric and magnetic fields in such a stripline with a slot at a distance z_n from O (Origin as shown in fig. 2.3) can be expressed as [4]

$$\mathbf{E}^+ = C_T\ \mathbf{e}_T(x,y)\ e^{-j\beta(z-z_n)} \quad \ldots\ldots\ldots\ 2.1$$

$$\mathbf{H}^+ = C_T\ \mathbf{h}_T(x,y)\ e^{-j\beta(z-z_n)} \quad \ldots\ldots\ldots\ 2.2$$

$$\mathbf{E}^- = B_T\ \mathbf{e}_T(x,y)\ e^{+j\beta(z-z_n)} \quad \ldots\ldots\ldots\ 2.3$$

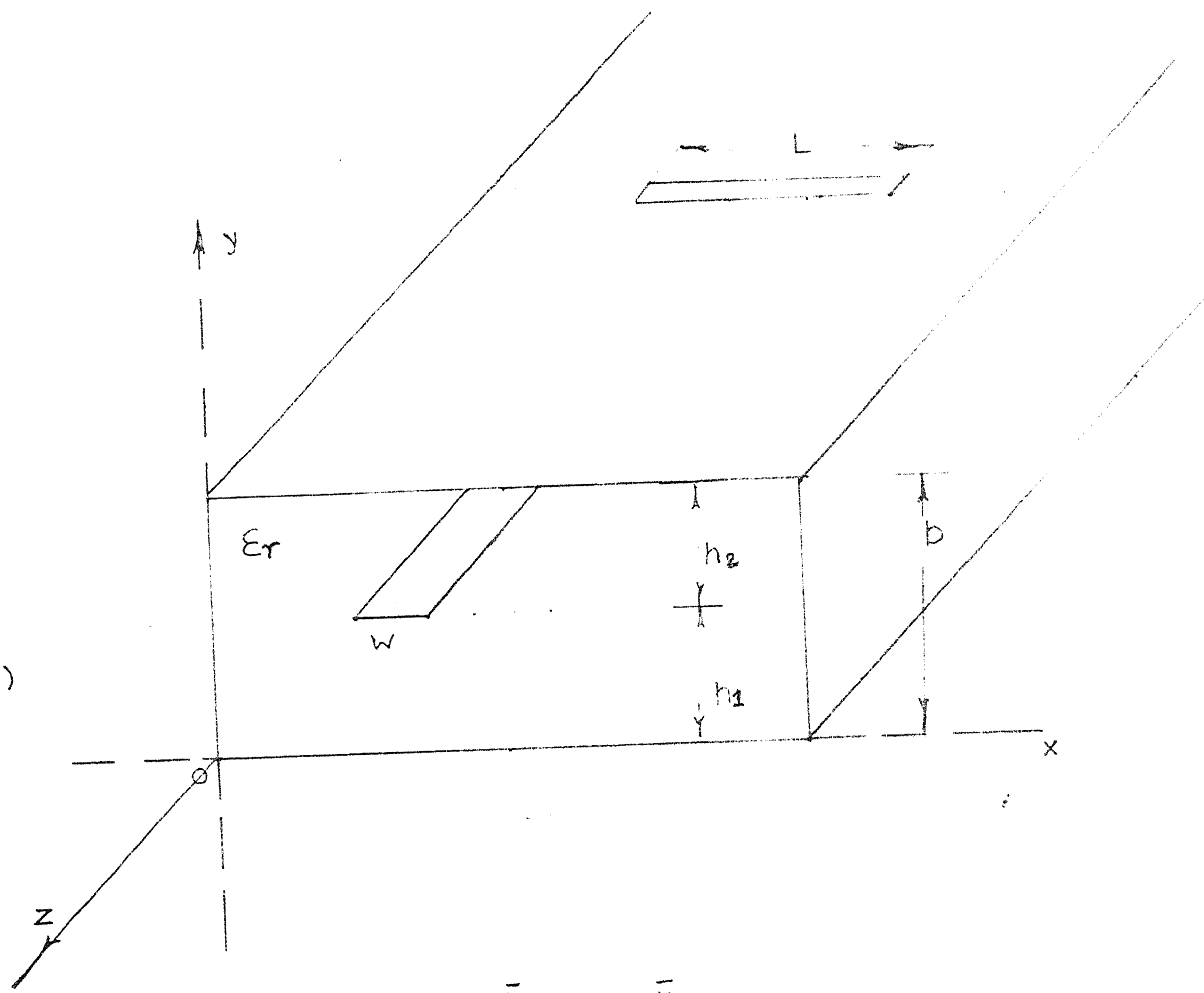
$$\mathbf{H}^- = -B_T\ \mathbf{h}_T(x,y)\ e^{+j\beta(z-z_n)} \quad \ldots\ldots\ldots\ 2.4$$

where **E** and H denote electric and magnetic fields and $\mathbf{e}_T$ (x,y) and $\mathbf{h}_T$ (x,y) are the normalized transverse electric and magnetic fields inside a stripline. Any quantity with a boldface type signifies a vector quantity. The superscript + and - denote regions $z>z_n$ and $z<z_n$ respectively. β is the propagation constant of the TEM mode. Since our assumption is that the slot is a thin slot $(d<<\lambda)$, the slot electric field distribution can be assumed to be of the form,

$$\begin{aligned}\mathbf{E}^S &= \mathbf{U}_z\ E^S_m\ \sin K(L_n/2 - |x|) \\ &= \mathbf{U}_z\ E^S_m\ g\ (x, L_n) \quad \ldots\ldots\ldots\ 2.5\end{aligned}$$

where E^S_m is the magnitude of the slot electric field at the centre of the slot, L_n is the length of the slot, the function

(a)



(b)

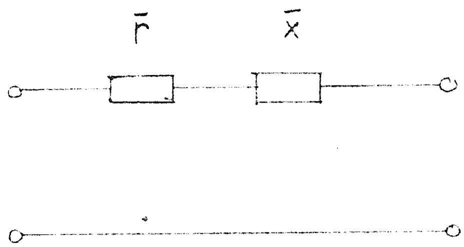


Fig 2.2 (a) Transverse slot on a boxed stripline.

(b) The equivalent circuit of a slot.

$$g\,(x,L_n) = \sin K(L_n/2 - |x|)\ldots\ldots\ldots\ldots\ldots\ldots 2.6$$

and, $\mathbf{U}_z$ is a unit vector in the z direction. Following the method given in [4], B_T and C_T are determined.

$$B_T = \tfrac{1}{2}\iint_{slot} n \times E^s.\mathbf{h}_T\ e^{-j\beta(z-z_n)}\ ds\ldots\ldots 2.7$$

and

$$C_T = -\tfrac{1}{2}\iint_{slot} n \times E^s.\mathbf{h}_T\ e^{-j\beta(z-z_n)}\ ds\ldots\ldots 2.8$$

where the integration is over the slot area, and **n** is the unit normal to slot **area pointing** toward the negetive y direction.

The discontinuity in the modal voltage is defined to be

$$\Delta V = B_T - C_T\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.9$$

$$= \iint_{slot} \mathbf{n} \times E^s.\mathbf{h}_T\ \cos\beta(z-z_n)\ ds\ldots\ldots\ldots 2.10$$

Integrating 2.7, 2.8 and 2.10 along z direction, we get

$$B_T = \tfrac{1}{2}\int_L V^s_n\ (\mathbf{n} \times \mathbf{U}_z.\mathbf{h}_T)\ g\,(x,L_n)\ dx\ldots\ldots\ldots 2.11$$

$$C_T = -\tfrac{1}{2}\int_L V^s_n\ (\mathbf{n} \times \mathbf{U}_z.\mathbf{h}_T)\ g\,(x,L_n)\ dx\ldots\ldots\ldots 2.12$$

$$\Delta V = V^s_n \int_L (\mathbf{n} \times \mathbf{U}_z.\mathbf{h}_T)\ g\,(x,L_n)\ dx\ldots\ldots\ldots 2.13$$

where $\quad V^s_n = E^s_m d\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.14$

is the maximum voltage across the slot, and d is the width of the slot.

Also from 2.11 and 2.12 we get

$$B_T = -C_T\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.15$$

The complex power in the slot is given by [1]

$$P = P_r + j\,P_i\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.16$$

Where P_r is the radiated power and P_i is the reactive power or power stored in the slot fields. The expressions for P_r and P_i are given by

$$P_r = \frac{V_n^2}{\eta\pi}\Big\{\mathrm{cin}(KL_n) + [\mathrm{cin}(KL_n) - \tfrac{1}{2}\,.\,\mathrm{cin}(2KL_n)]\cos(KL_n) - [\mathrm{si}(KL_n) - \tfrac{1}{2}\,\mathrm{si}\,(2KL_n)]\ \sin\,(KL_n)\Big\}\ \ldots\ldots\ldots\ldots 2.17$$

$$P_i = \frac{V_n^2}{\eta\pi}\Big\{\mathrm{si}\,(KL_n) + [\mathrm{si}(KL_n) - \tfrac{1}{2}\,\mathrm{si}(2KL_n)]\ \cos(KL_n) + [\mathrm{cin}(KL_n) - \tfrac{1}{2}\,\mathrm{cin}(2KL_n) - \ln(e^{3/2}\ L_n/2d)]\ \sin\,(KL_n)\Big\}\ \ldots\ldots 2.18$$

where

$$\mathrm{Si}(x) = \int_0^x \sin t'/t'\ \ dt' \text{ and } \mathrm{cin}(x) = \int_0^x (1-\cos t')/t'\ \ dt'$$

$V_n = V^s_n$, the maximum voltage across the slot,

is the equivalent free space wave impedance with the effective dielectric constant ε_r throughout the space (inside as well as outside the stripline).

$$K = \frac{2\pi}{\lambda_0}\sqrt{\varepsilon_r'}\ \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.20$$

$$= \beta$$

ε_r' is the relative permittivity of the uniform medium replacing the two different half spaces on either side of the aperture and is given by

$$\varepsilon_r' = \frac{2\varepsilon_r}{1+\varepsilon_r} \quad \text{.............................2.21}$$

where ε_r is the permittivity of the dielectric used in the stripline.

Since there are resistive and reactive powers associated with the slot, it is appropriate to define an equivalent impedance. Let us see the nature of the slot from the circuit point of view.

There is a lumped series impedance Z at $z=z_n$ in a transmission line of characteristic impedance Z_o, as shown in fig. 2.5. The line is terminated in both directions. An electromagnetic wave of type $A\ e^{-j\beta(z-z_n)}$ is incident from -z direction, and the reflected and transmitted scattered waves are $B\ e^{+j\beta(z-z_n)}$ and $C^{-j\beta(z-z_n)}$. the transmission line equations are,

$$V(z_n^-) = A\ e^{-j\beta(z-z_n)} + B\ e^{+j\beta(z-z_n)} \quad \text{............2.22}$$

$$I(z_n^-) = \frac{A}{Z_o}\ e^{-j\beta(z-z_n)} - \frac{B}{Z_o}\ e^{+j\beta(z-z_n)} \quad \text{...........2.23}$$

$$V(z_n^+) = (A+C)\ e^{-j\beta(z-z_n)} \quad \text{.....................2.24}$$

$$I(z_n^+) = \left(\frac{A+C}{Z_o}\right)\ e^{-j\beta(z-z_n)} \quad \text{.....................2.25}$$

where V(X) and I(X) are voltage and currents at z = X, the superscript + is for the region $z > z_n$ and the superscript - is for the region $z < z_n$. β is the propagation constant of the wave. Since Z is a series impedance,

$$I(z_n^-) = I(z_n^+) = I(z_n) \quad \text{.......................2.26}$$

and
$$V(z_n^-) = V(z_n^+) + I(z_n)\ Z \quad \text{....................2.27}$$

Substituting 2.23 and 2.25 into 2.26 we obtain

$$B = -C \quad \text{.................................2.28}$$

Comparing 2.28 and 2.15 we say that the thin transverse slot acts as a series impedance to the strip transmission line.

The slot impedance Z, normalized with respect to the characteristic impedance Z_o of the strip transmission line is given by [1]

$$Z = \frac{\Delta V^2}{Z_o P} \quad \text{.................................2.29}$$

Where ΔV is the discontinuity in modal voltage and P is the complex power in the slot.

The above equation highlights a very important point. Since complex power does not depend on the co-ordinate system used and Z is also an invariant quantity with respect to the change of co-ordinate system, ΔV must also be an invariant quantity with respect to a change in the co-ordinate system. Now, we shift our axis as shown in fig. 2.2 for evaluation of ΔV.

Following Das & Prasad [1] once again,

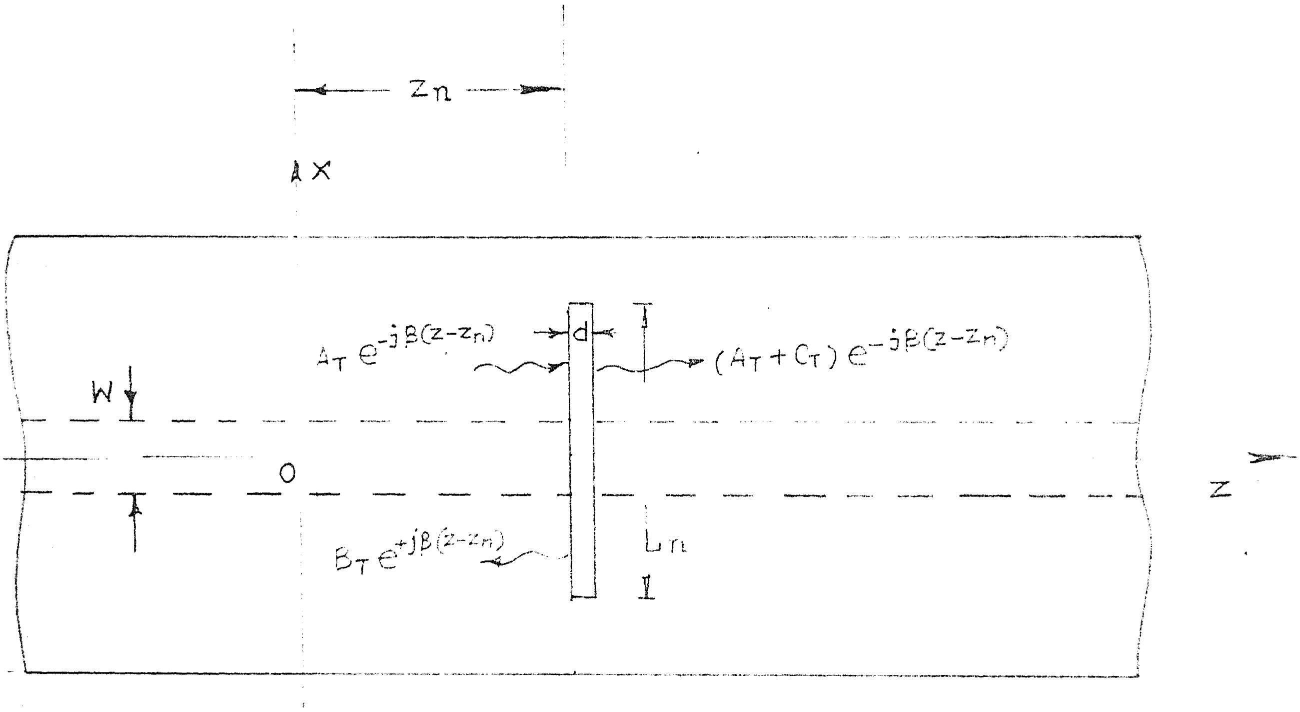


Fig 2.3. The incident, transmitted and the reflected waves from a slot.

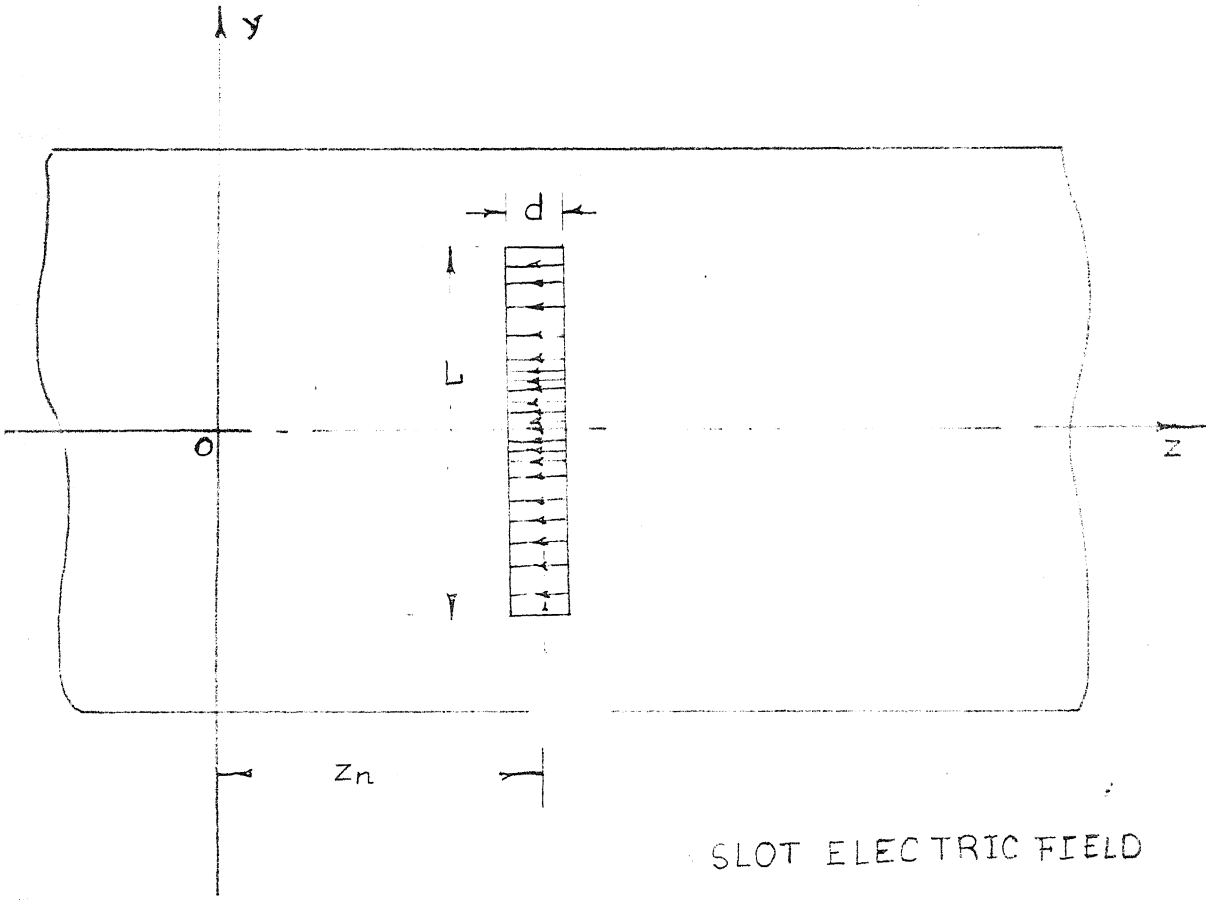


Fig 2.4. The electric field distribution in a thin ($d<<\lambda$) slot.

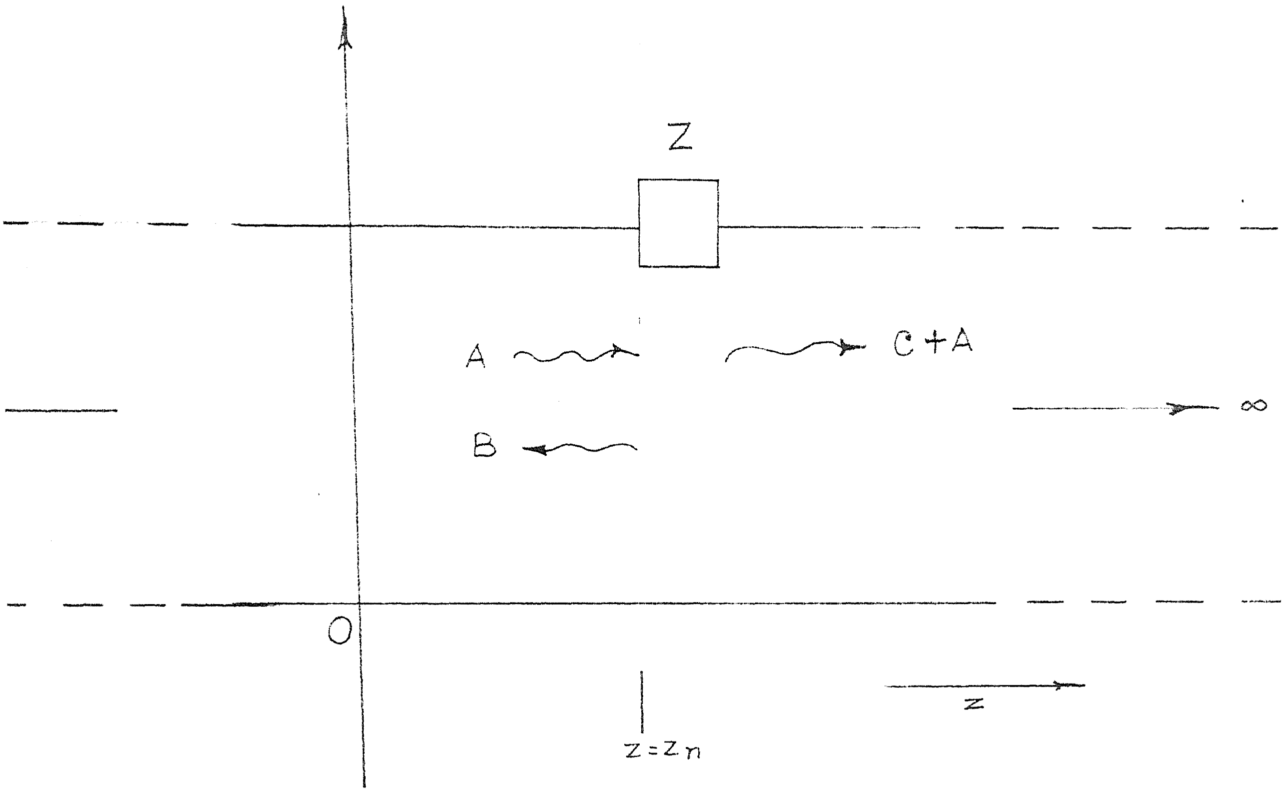


Fig 2.5. Equivalent transmission line loaded with a lumped series impedance for a stripline-fed slot.

$$\Delta V = V_n \sum_{i=1,3,5}^{\infty} \frac{\sqrt{2\pi}}{aR(\Delta i)} J_0\left(\frac{i\pi\omega}{2a}\right) \sinh\left(\frac{i\pi h_1}{a}\right) \cdot$$

$$\cdot K \left[\frac{\cos\left(\frac{KL_n}{2}\right) - \cos\left(\frac{i\pi L_n}{2a}\right)}{\left(\frac{i\pi}{a}\right)^2 - K^2} \right] \quad \text{.........2.30}$$

where

$$R^2 = \sum_{i=1,3,5}^{\infty} \frac{1}{i} \frac{J_0^2\left(\frac{i\pi\omega}{2a}\right)\left[\coth\left(\frac{i\pi h_1}{a}\right) + \coth\left(\frac{i\pi h_2}{a}\right)\right]}{\left[\varepsilon_r' \coth\left(\frac{i\pi h_1}{a}\right) + \varepsilon_r' \coth\left(\frac{i\pi h_2}{a}\right)\right]^2} \quad \text{.........2.31}$$

$$\Delta i = \varepsilon_r' \sinh\left(\frac{i\pi h_2}{a}\right) \cosh\left(\frac{i\pi h_1}{a}\right) + \varepsilon_r' \cosh\left(\frac{i\pi h_2}{a}\right) \sinh\left(\frac{i\pi h_1}{a}\right) \quad \text{.........2.32}$$

and $J_0(x)$ is the Bessel function of order zero. From equations 2.11 and 2.13,

$$B_T = \tfrac{1}{2} \Delta V \quad \text{.................................2.33}$$

and from equations 2.15 and 2.33,

$$C_T = -\tfrac{1}{2} \Delta V \quad \text{.................................2.34}$$

Simplifying the expression for V we get from 2.33

$$B_T = V_n \; f_n \quad \text{.............................2.35}$$

where $f_n = \tfrac{1}{2} \Delta V / V^s_n$ 2.36

Also since after $z = z_n$, at the right of the slot, the line extends to +, or matched to Z_o, we can find the reflection co-efficient Γ due to the slot impedance.

$$\Gamma = B_T/A_T \quad \text{.......................... 2.37}$$

Looking from the left of the slot, the input impedance is

$$Z_{in} = Z_n + Z_o$$

where Z_n is the equivalent slot impedance.

The reflection co-efficient can also be expressed as

$$\Gamma = \frac{(Z_o + Z_n) - Z_o}{(Z_o + Z_n) + Z_o} \quad \text{.............................. 2.38}$$

Comparing 2.38 and 2.37 we obtain

$$Z_n/Z_o = 2B_T/(A_T - B_T) \quad \text{.........................2.39}$$

Simplifying further and substituting the expression for B_T from 2.35,

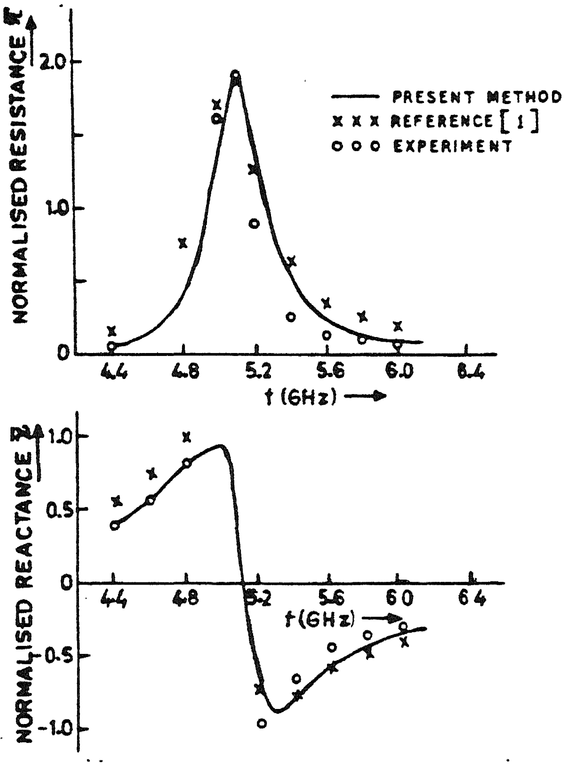


Fig 2.6. Reproduced from [1]. Normalized impedance versus frequency.

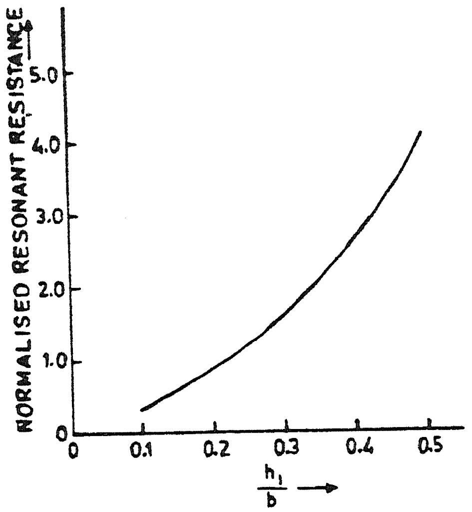


Fig 2.7. Reproduced from [1]. Normalized resistance versus h_1/b.

$$\frac{V_n}{A_T} = \left(\frac{\frac{Z_n}{Z_o}}{2 + \frac{Z_n}{Z_o}} \right) \cdot \frac{1}{f_n} \qquad \text{.........................2.40}$$

This is an important result which will be used later.

2.3. RESULTS AND DISCUSSION :

The plot of equivalent series impedance normalized to the characteristic impedance of the line Z_0, is shown in fig. 2.6. As the frequency is varied from 4.4 to 6.4 GHz the normalized resistance rises from a very low value (near zero) to a very high value, 2.0 and then once again goes down from 2.0 to a very low value. It is almost symmetric about a frequency (5.06667 GHz), at which the resistance is 2.0. The reactance is seen to be inductive below this frequency and capacitive above it. At 5.06667 GHz the reactance is zero. This frequency is called the resonant frequency of the slot. Since the length of the slot used is 2.4 cm., the L/λ_{res} is found to be 0.405336.

Also from fig. 2.7, normalized resonant resistance versus h_1/b plot shows that the normalized resonant resistance increases in a nonlinear way as h_1/b increases.

For a balanced stripline with $h_1 = h_2$, $h_1/b = 0.5$ the normalized resonant resistance is quite high; around 4.5. This indicates that to make an array of slots on the upper ground plane of a balanced stripline of characteristic impedance 50 ohms, it may be very difficult to find an input match where at least one slot impedance is close to 4.5.

Since h_1 can be decreased in steps only because of the availability of substrates in standard thicknesses, h_1/b is considered as 1/3 and the resonant resistance is found to be 2.0. This value is much easier to match to a standard 50 ohms line.

CHAPTER 3

ANALYSIS OF MUTUAL COUPLING BETWEEN SLOTS

3.1. INTRODUCTION AND THE METHOD OF ANALYSIS

In an array of slots there is bound to be mutual coupling between the slots, the coupling factor increasing with decreasing slot separation. When there is mutual coupling between the slots, the slot voltage is different from the value it would have when it is isolated. In this analysis the receiprocity theorm has been made use of in the calculation of mutual coupling. The receiprocity theorem considers two situations to calculate the induced voltage at a particular slot due to the presence of other slots. The two situations are:

Situation 'a'. This situation consists of only one slot, at the position of the n^{th} slot, which is radiating and all other slots are short circuited or covered by conducting sheet.

Situation 'b'. All the slots other than the n^{th} one are acting as sources and the n^{th} slot is acting as a receiver or connected to a matched load. In each of the above cases the fields inside as well as outside the stripline are evaluated. Then, the expression for fields are substituted into the receiprocity relationship and the coupling voltage V^s (the maximum voltage at the centre of the slot) is determined. Also, it is assumed that the form of the slot electric field distribution remains same as in an isolated slot i.e., sinusoidal $(\sin k(L/2 - |x|))$, because the slot width is very small with respect to the wavelength of operation. The above analysis follows the procedure as given by Elliott [3].

3.2. ANALYSIS :

The first step is to invoke the Receiprocity theorem. If the electric and magnetic source current distributions $\bar{J}^a$, $\bar{J}^a_m$ produce fields $\bar{E}^a$, $\bar{H}^a$ and another set of sources $\bar{J}^b$, $\bar{J}^b_m$ produce fields $\bar{E}^b$ and $\bar{H}^b$, then according to the receiprocity theorem the following equality holds.

$$\int_V (E^b.J^a - \mu_o H^b.J^b_m)\, dV = \int_V (E^a.J^b - \mu_o H^a.J^b_m)\, dV \quad ......3.1$$

where V is the volume enclosing the sources and as shown in fig. 3.2.

Consider the coupled slot geometry shown in fig. 3.1. Two slots of lengths L_n and L_m are shown situated at $z = z_n$ and $z = z_m$ respectively. They are placed evenly on both sides of the centre strip. The reason for showing

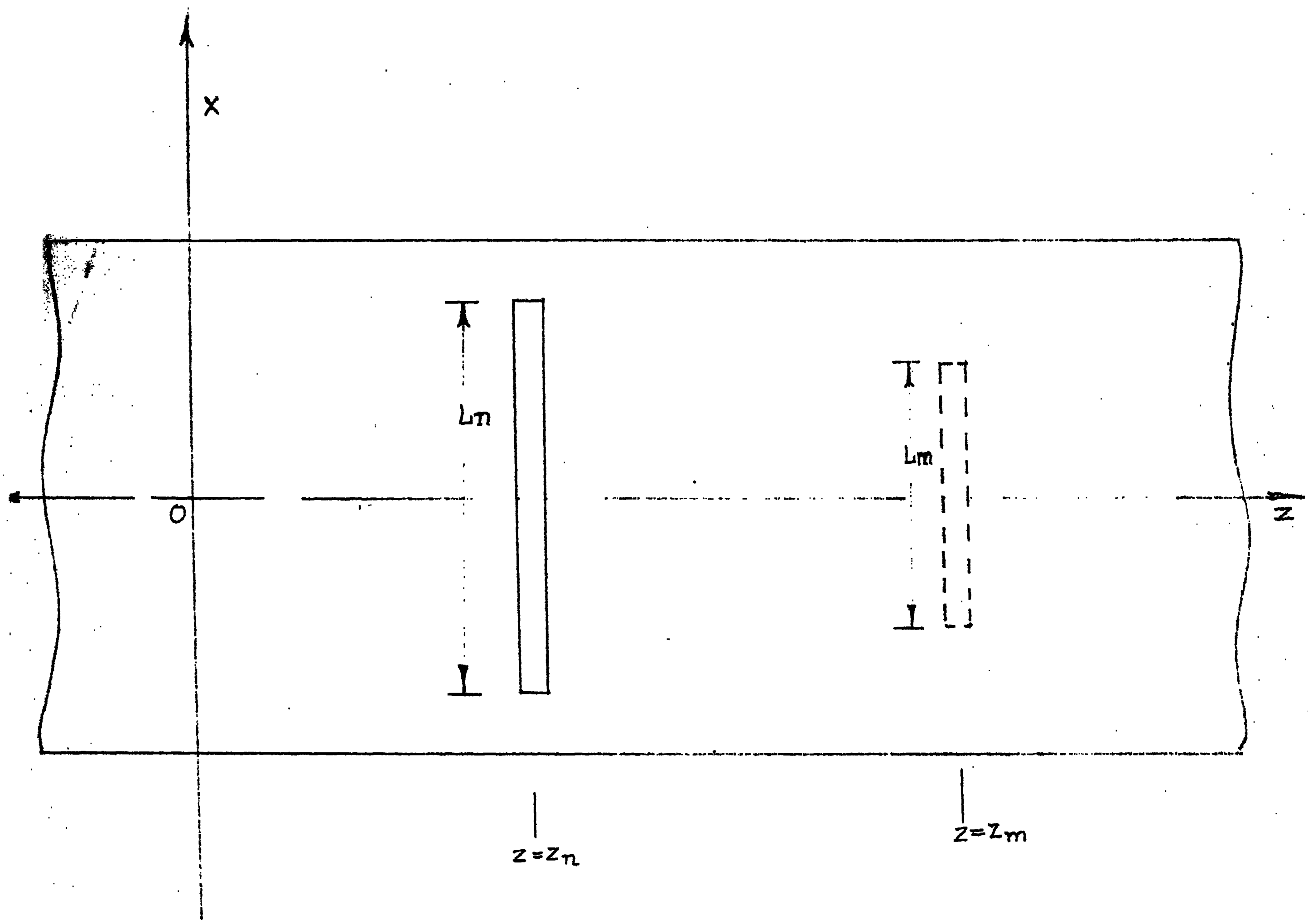


Fig 3.1. Coupled slot geometry.

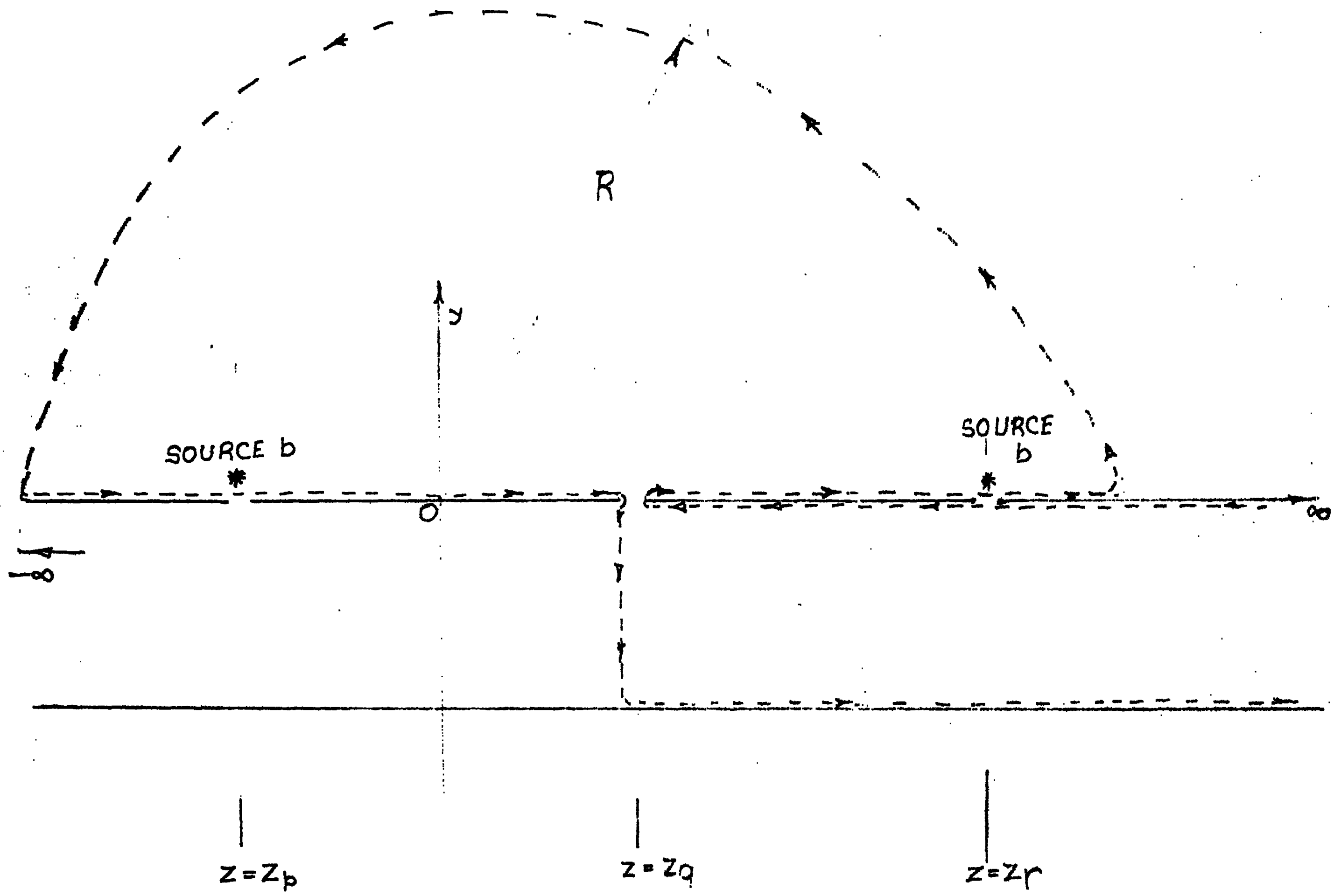


Fig 3.2. - Volume of integration for evaluation Receiprocity integral.
R tends to ∞. Volume is inside the dotted region.

the m^{th} slot in dotted lines will be explained later.

Situation 'a': In the situation 'a', the n^{th} slot is the only slot present on the ground plane. Let a TEM mode, of amplitude A^a, be incident on the n^{th} slot from -z direction. A slot voltage $V^{s,a}_n$ develops at the centre of the n^{th} slot, and TEM waves of amplitude B^a and C^a are scattered by the slot. The TEM fields in the $z < z_n$ region are,

$$\bar{H}^a_x = \mathbf{h}_T \quad [-A^a \, e^{-j\beta(z-z_n)} + B^a \, e^{+j\beta(z-z_n)}] \ldots\ldots\ldots.3.2$$

$$\bar{E}^a_y = \mathbf{e}_T \quad [A^a \, e^{-j\beta(z-z_n)} + B^a \, e^{j\beta(z-z_n)}] \ldots\ldots.3.3$$

where $\mathbf{h}_T$ and $\mathbf{e}_T$ are normalized TEM mode fields in a stripline [see equations 2.1 to 2.4]. We shall replace the fields on the plane $z = z_1$ where $z_1 < z_n - d/2$, by their equivalent sources.

According to the field equivalance principle, this will make all the fields and sources in the region $z<z_1$ zero, without affecting fields either in the region $z>z_1$ or in the half space outside the slot. These equivalent sources are given by

$$K^a = U_z \times H^a \ldots\ldots\ldots\ldots\ldots\ldots.3.4$$

$$K^a_m = -\mu_o^{-1} \, U_z \times E^a \ldots\ldots\ldots.3.5$$

where K^a and K^a_m are the equivalent electric and magnetic current sources.

Substituting equations 3.2 and 3.3 in 3.4 and 3.5 we have

$$|K^a| = \quad |\mathbf{h}_T| \; [-A^a \, e^{-j\beta(z-z_n)} + B^a \, e^{j\beta(z-z_n)}] \ldots\ldots.3.6$$

$$|K^a_m| = \quad |\mathbf{e}_T| \; [A^a \, e^{-j\beta(z-z_n)} + B^a \, e^{j\beta(z-z_n)}] \ldots\ldots3.7$$

All the other sources of situation 'a' are electric type and flow on the ground plane. As in the situation 'a' it is assumed that the stripline is matched in the region $z>z_n+d/2$ and since in equation 3.1 $\mathbf{E}^b_T$ will be zero at all such points, these sources need not be identified.

In the situation 'b' once again, only the n^{th} slot will be present, but there will be no sources at $z = -\infty$ to cause an incident TEM wave. Instead, the sources will be magnetic current sheets placed on the outer half space,

skin tight against the ground plane, lying in areas whose projections on the ground plane correspond to the regions occupied by each of the other slots in the actual array. These magnetic sources will be chosen so as to contribute to the external field precisely as would the actual electric fields in the actual slots. They will excite the n^{th} slot externally, the result being that a TEM wave of amplitude B^b propagate in the -z direction in the stripline containing the n^{th} slot. This wave passes through the cross-section $z = z_1$. When the receiprocity theorem is applied to this pair of situations, we find that,

$$\int_{s1} (\mathbf{E}^b.\mathbf{K}^a - \mu_o \mathbf{H}^b.\mathbf{K}^a{}_m)\, dS_1 = -\sum_{m=1}^{N}{}' \int_{sm} \mu_o \mathbf{H}^a{}_{ext}.\mathbf{K}^b{}_{m\ ext}\, dS_m \quad ..3.8$$

where S_1 is the stripline cross-section at $z = z_1$

S_m is the surface area normally occupied by the m^{th} slot. The prime (′) on the summation sign means that the term m = n is excluded.

The components of the TEM mode of amplitude B^b are given by,

$$\bar{H}^b = B^b\, \mathbf{h}_T\, e^{j\beta(z-z_n)} \quad ....3.9$$

$$\bar{E}^b = B^b\, \mathbf{e}_T\, e^{j\beta(z-z_n)} \quad ....3.10$$

By substituting equations 3.9, 3.10, 3.6 and 3.7 into the L.H.S of 3.8,

$$\int_{s1} (\mathbf{E}^b.\mathbf{K}^a - \mu_o \mathbf{H}^b.\, \mathbf{K}_m)\, dS_1 = -2A^a\, B^b \quad .....3.11$$

Equating the L.H.S. and R.H.S. of 3.8, we obtain

$$B^b = \frac{1}{2A^a} \sum_{m=1}^{N}{}' \int_{sm} \mu_o \mathbf{H}^a{}_{ext}.\, \mathbf{K}^b{}_{ext}\, dS_m \quad ....3.11$$

Now, we will have to find out $\mathbf{H}^a{}_{ext}$ along the ground plane. The electric field distribution in the n^{th} slot is assumed to be in the form

$$\mathbf{E}^s = \mathbf{U}_z\, V^{s,a}{}_n (\sin K(L/2 - |x|))/d \; .$$

The fields produced in the half space by this slot excitation are the same as those of doubled magnetic current sheet

$$\mathbf{K}^a{}_m = \mathbf{U}_x\, 2\mu_o^{-1}\, V^{s,a}{}_n\, (\sin K(L/2 - |x|))/d \quad ....3.13$$

where U_x is a unit vector along x direction.

Let us construct a local co-ordinate system (x, η, ζ) with origin at the centre of the n^{th} slot.

The electric vector potential function due to this doubled magnetic current sheet has only an x component, given by

$$F_x(x, y, \zeta) = \int_{-\frac{L_n}{2}}^{\frac{L_n}{2}} \int_{-\frac{d_n}{2}}^{\frac{d_n}{2}} \left(\frac{2\mu_0^{-1} V_x^{s,a}}{d_n}\right) \frac{e^{-jKR}}{4\pi\mu_0^{-1}R} \sin K\left(\frac{L_n}{2} - |x|\right) dx' \, d\zeta$$

$$= \frac{V^{s,a}{}_n}{2\pi} \int_{-\frac{L_n}{2}}^{\frac{L_n}{2}} \sin K\left(\frac{L_n}{2} - |x|\right) \frac{e^{-jKR}}{R} dx' \qquad \text{......... 3.14}$$

where $R = \sqrt{(x_n - x_n')^2 + \eta^2 + \zeta^2}$ 3.15

is the distance from a point (x_n', o, o) on the axis of the n^{th} slot to an arbitrary point (x_n, η, ζ) measured in local co-ordinates.

Because $jw\mu_0 H = \nabla X \nabla X$ **F**, it follows that

$$H^a_{x,ext}(x_n, \eta, \zeta) = \frac{1}{j\omega\mu_0}\left(\frac{\partial^2}{\partial x_n^2} + K^2\right) F_x(x_n, \eta_n, \zeta_n)$$

$$= \frac{V^{s,a}{}_n}{2\pi j\omega\mu_0} \int_{-\frac{L_n}{2}}^{\frac{L_n}{2}} \sin\left(\frac{L_n}{2} - |x_n'|\right)\left[\frac{\partial^2}{\partial x_n^2} + K^2\right] \frac{e^{-jKR}}{R} dx_n' \qquad \text{....... 3.16}$$

Since $K^b{}_{m,ext} = \hat{U}_z(\mu_0^{-1} V_m{}^{s,b}/d) \sin K(L_m/2 - |x_m'|)$ in terms of local co-ordinates at the m^{th} slot, if we combine equations 2.12 and 3.16 the expression for B^b becomes,

$$B^b = \frac{1}{4\pi j\omega\mu_0} \cdot \left(\frac{V^{s,a}{}_n}{A^a}\right) \sum_{m=1}^{N}{}' V_m{}^{s,b} \int_{-\frac{L_m}{2}}^{\frac{L_m}{2}} \sin K\left(\frac{L_n}{2} - |x_m'|\right)$$

$$\cdot \left\{ \int_{-\frac{L_n}{2}}^{\frac{L_n}{2}} \sin K\left(\frac{L_n}{2} - |x_n'|\right)\left[\frac{\partial^2}{\partial x_n^2} + K^2\right] \frac{e^{-jKR}}{R} dx_n \right\} dx_m \qquad \text{...... 3.17}$$

where $K^2 = \omega^2 \mu_0 \varepsilon_r' \varepsilon_0$ and ε_r' is the equivalent dielectric constant of the whole space (see equation 2.21). And R is the distance from a point (x_m', o, o) to a point (x_n', o, o).

From 2.40 we obtain relation of $V^{s,a}{}_n/A^a$, and $V_m^{s,b}$ is the actual slot voltage in the m^{th} slot, and so we can now delete the superscript b from that quantity. Finally B^b can be related to the slot voltage $V^s{}_n$, induced at the n^{th} slot due to the mutual coupling in the form,

$$B_b = f_n V^s_n, \text{ mutual [from 2.35]}$$

Subtituting this expression of B^b in equation 3.17, we arrive at the following expression for the slot voltage due to mutual coupling.

$$V^s_{n,\text{mutual}} = \frac{1}{4\pi j\omega\mu_0} \cdot \left(\frac{\frac{Z_n}{Z_0}}{2+\frac{Z_n}{Z_0}}\right) \frac{1}{f_n^2} \sum_{m=1}^{N}{}' V^s_m P_{mn} \qquad \ldots\ldots 3.18$$

where
$$P_{mn} = \int_{-\frac{L_m}{2}}^{\frac{L_m}{2}} \sin K\left(\frac{L_m}{2} - |x'_m|\right) \left\{ \int_{-\frac{L_n}{2}}^{\frac{L_n}{2}} \sin K\left(\frac{L_n}{2} - |x'_n|\right) \cdot \left[\frac{\partial^2}{\partial x_n^2} + K^2\right] \frac{e^{-jKR}}{R}\, dx'_n \right\} dx'_m \qquad \ldots\ldots 3.19$$

With two integrations by parts the above expression reduces to,

$$P_{mn} = K \int_{-\frac{L_m}{2}}^{\frac{L_m}{2}} \sin K\left[\frac{L_m}{2} - |x'_m|\right] \cdot \left[\frac{e^{-jKR_1}}{R_1} + \frac{e^{-jKR_2}}{R_2} - 2\cos\left(K\frac{L_n}{2}\right)\frac{e^{-jKR}}{R}\right] \qquad \ldots\ldots 3.20$$

where
$$R_1 = \sqrt{(x'_m - L_n/2)^2 + z_{mn}^2}$$
$$R_2 = \sqrt{(x'_m + L_n/2)^2 + z_{mn}^2}$$
$$R = \sqrt{x'^2_m + z_{mn}^2}$$

z_{mn} is the distance between m^{th} and n^{th} slots and K is propagation constant of the TEM wave [see equation 2.20].

3.3. DISCUSSION

The expression for P_{mn} is a familiar one which occurs in the calculation of mutual impedance between two unequal dipoles of lengths L_m and L_n. Since by receiprocity theorem, the mutual impedance seen by the antenna with length L_m and the mutual impedance seen by the antenna with length L_n are the same, interchanging of L_m by L_n in the expression for P_{mn} should not alter the value of P_{mn}. In otherwords $P_{mn} = P_{nm}$.

CHAPTER 4

ANALYSIS OF ARRAY OF SLOTS

4.1. INTRODUCTION/METHOD OF ANALYSIS :

To analyse slot arrays one has to determine the factors contributing to the slot voltage V^S in every slot in an array. The form of the electric field in the slot is assumed to be once again, sinusoidal as in the previous chapter. The slot voltage V^S in a given slot is a superposition of three voltages,

1. due to a wave incident from the left handside of the slot,
2. due to a wave incident from the right hand side of the slot, and
3. due to mutual coupling between this slot and other slots of the array.

Once these three components are known, they are added to obtain V^S.

In the absence of mutual coupling, the impedance of the slot is purely the self impedance. But with the presence of other slots, the effective impedance of a given slot gets modified. In the following analysis, the slot voltage is substituted in equation 2.35 to obtain the back scatter co-efficient. This in turn can be related to an equivalent series impedance in the strip transmission line. By this procedure we can convert a slot array fed by a stripline into an equivalent transmission line loaded with series impedances as given by the equivalent impedance of the slots. With this equivalent circuit, the equivalent impedance of the slot array as seen at the feed point is also derived.

Also it is to be remembered that whenever and whereever we refer to the slot impedance or effective slot impedance, it is to be understood as the impedance faced by the stripline due to the module containing the slot on the upper groundplane surrounded by pin curtains. Finally at the end of this chapter an iterative procedure is developed to calculate the effective slot impedance of a single slot in an array.

4.2. ANALYSIS OF THE ARRAY :

In figure 4.1 the slot is shown enclosed by two cross-sectional planes perpendicular to the ground plane. One is at a distance of $z_n - \varepsilon/2$ and the other $z_n + \varepsilon/2$. ε is slightly greater than the slot width, just sufficient to place the pin curtains. This region bounded by the surfaces forms one module. There are N such modules corresponding to N slots. In any module, A_T and D_T are the incident

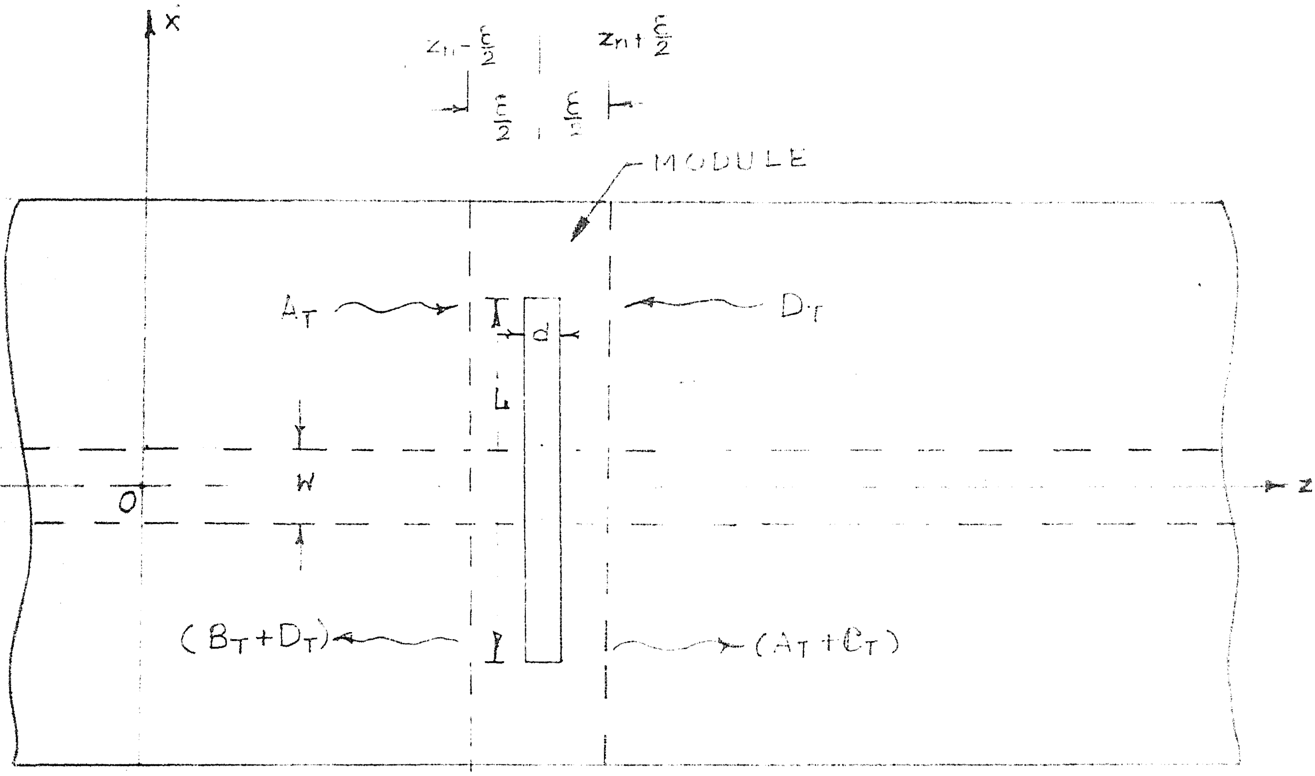


Fig 4.1. The basic array module.

waves from the left and the right of the module respectively. B_T and C_T are the amplitudes of the scattered waves. The form of the slot electric field is

$$\mathbf{E}^s_n = \mathbf{U}_z V^s_n (\sin K(L_n/2 - |x_n|))/d$$

but the voltage V^s_n is now comprised of three components; one is due to the incident wave from the left, one from the right and the voltage induced due to the mutual coupling of N-1 other slots. Here A_T is the incident wave from the left due to the cumulative effect of all slots. So is the case with D_T. Therefore, if we assume that all the other slots are absent except the one at $z = z_n$ and incident waves from the left and the right are A_T and B_T then the situation remains same, as far as the slot is concerned. This analysis is exactly the same as we have done before in chapter 2.2.

Substituting V^s_n as a superposition of three voltage components in equations 2.35, we get

$$B_T = (V^s_{n,1} + V^s_{n,2} + V^s_{n,3}) f_n \quad \text{.......................4.1}$$

where $V^s_{n,1}$ is due to a TEM wave of amplitude A_T incident from the left when all other slots are covered with conducting tapes and the stripline is terminated in a matched load at the right of the slot,

$V^s_{n,2}$ is due to a TEM wave of amplitude D_T incident from the right when all the other slots are shorted and the stripline is terminated in a matched load at the left of the module,

and $V^s_{n,3}$ is due to external mutual coupling with other N-1 slots in the array.

From 2.40 we get, $V^s_{n,1} = \left(\dfrac{\frac{Z_n}{Z_o}}{2 + \frac{Z_n}{Z_o}} \right) \dfrac{A_T}{f_n}$ 4.2

Similarly $V^s_{n,2} = -\left(\dfrac{\frac{Z_n}{Z_o}}{2 + \frac{Z_n}{Z_o}} \right) \dfrac{D_T}{f_n}$ 4.3

And from 3.18, $V^s_{n,3} = \dfrac{1}{4\pi j\omega\mu_o} \cdot \left(\dfrac{\frac{Z_n}{Z_o}}{\frac{Z_n}{Z_o}} \right) \cdot \dfrac{1}{f_n^2} \sum_{\substack{m=1 \\ m\neq n}}^{N}{}' V^s_m P_{mn}$ 4.4

where Z_n is the self impedance of the n^{th} slot.

Adding $V^s_{n,1}$, $V^s_{n,2}$ and $V^s_{n,3}$ we obtain,

$$A_T - D_T = V^s_n f_n \left(\frac{\frac{Z_n}{Z_o}}{2 + \frac{Z_n}{Z_o}} \right) - \frac{\sum_{m=1}^{N}{}' V^s_m P_{mn}}{4\pi j\omega\mu_o f_n} \quad \text{.................. 4.5}$$

Where (′) on the summation sign implies that the n^{th} slot is not included.

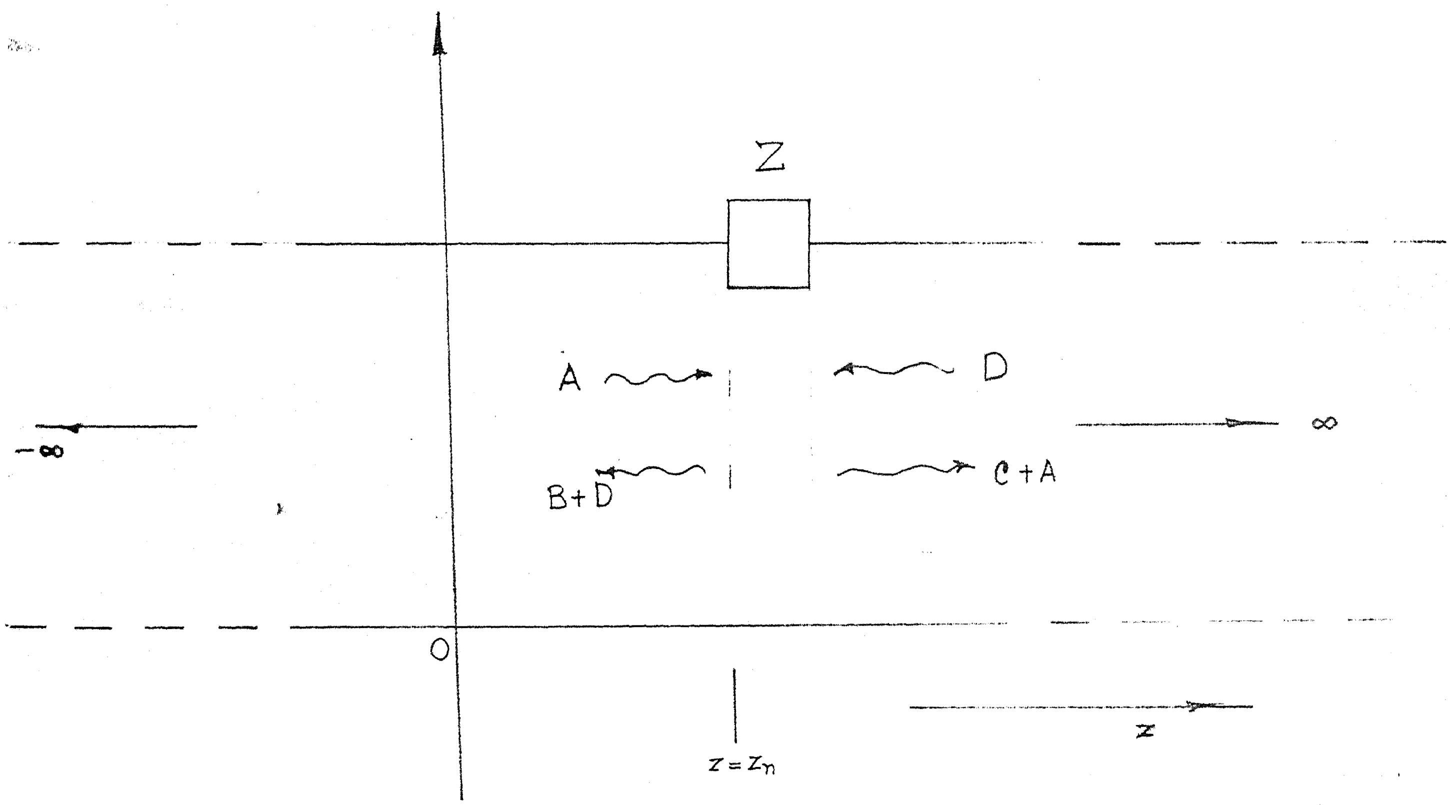


Fig 4.2. Equivalent circuit of an array module.

Now, consider the equivalent circuit of a module as shown in fig. 4.2. Here once again, A and D are the incident waves from the left and the right of the lumped impedance Z and B and C are the scattered waves, going towards -ve and +ve z directions respectively.

For $z<z_n$ the transmission line equations are

$$V(z) = A\, e^{-j\beta(z-z_n)} + (D + B)\, e^{j\beta(z-z_n)} \quad \text{.......... 4.6}$$

$$I(z) = \frac{A}{Z_o} e^{-j\beta(z-z_n)} - \frac{(D+B)}{Z_o} e^{j\beta(z-z_n)} \quad \text{.......... 4.7}$$

And for $z>z_n$, $V(z) = (A + C)\, e^{-j\beta(z-z_n)} + D\, e^{j\beta(z-z_n)}$ 4.8

$$I(z) = \left(\frac{A+C}{Z_o}\right) e^{-j\beta(z-z_n)} - \frac{D}{Z_o} e^{j\beta(z-z_n)} \quad \text{.......... 4.9}$$

The boundary conditions are :

$$I(z_n^-) = I(z_n^+) = I(z_n) \quad \text{.......... 4.10}$$

$$V(z_n^-) = Z^a_n\, I(z_n) + V(z_n^+) \quad \text{.......... 4.11}$$

Substituting 4.7 and 4.9 in 4.10 we get as before

$$B = -C$$

and substituting 4.6 and 4.8 in 4.11

$$B = \tfrac{1}{2} I_n Z^a_n \quad \text{.......... 4.12}$$

i.e. I_n is the mode current at $z = z_n$ and $I_n = I(z_n)$

This B and B_T could be related if they have the same phase at any cross-section z and their powers are the same in both the cases.

Therefore, $|B_T|^2 \int_s \mathbf{E}_T \times \mathbf{H}_T^* . \mathbf{U}_z \, ds = |VI^*|$ 4.13

Where $\mathbf{E}_T = \mathbf{e}_T\, e^{j\beta(z-z_n)}$ and,

$\mathbf{H}_T = \mathbf{h}_T\, e^{j\beta(z-z_n)}$.

Taking cross product between E_T and $\mathbf{H}_T$ and integrating over the cross-section of the stripline.

$$\int_S \mathbf{E}_T \times \mathbf{H}_T^* \cdot U_z \, ds = -1$$

Also from equations 4.8 and 4.9, we have

$$V \, I^* = - B^2/Z_0$$

Therefore, equation 4.13 reduces to

$$B_T = B/\sqrt{Z_0} \qquad \ldots\ldots 4.14$$

Substituting 2.35 and 4.12 in 4.14 we obtain

$$Z^a_n = (2 \sqrt{Z_0} \, V^s_n \, f_n)/I_n \qquad \ldots\ldots 4.15$$

Again from fig. 4.2 we notice that at $z = z_n^-$ the input inpedance looking from the left, $Zeq_1 = V(z_n^-)/I(z_n^-)$

$$= Zo \; (A+D+B)/(A-D-B) \qquad \ldots\ldots 4.16$$

and the input impedance at $z = z_n^+$, $Zeq_2 = V(z_n^+)/I(z_n^+)$

$$= Z_0 \; (A+C+D)/(A+C-D), \qquad \ldots\ldots 4.17$$

and the difference between Zeq_1 and Zeq_2 gives the equivalent slot impedance Z^a_n.

$Z^a_n = Zeq_1 - Zeq_2$, which simplifies to

$Z^a_n/Z_0 = 2 \, B/(A-B-D)$ and

Considering the equivalance between the strip in transmissionline with a slot and a transmission line loaded with an equivalent lumped impedance, the expression for Z^a_n/Z_0 becomes,

$$Z^a_n/Z_0 = 2B_T/(A_T-B_T-D_T) \qquad \ldots\ldots 4.18$$

From 4.3 substituting the value of A_T-D_T and from 2.35 the above equation simplifies to

$$\frac{Z^a_n}{Z_0} = \frac{2 f_n}{f_n\left(\dfrac{2+\frac{Z_n}{Z_0}}{\frac{Z_n}{Z_0}}\right) - \dfrac{\Sigma' V^s_m P_{mn}}{4\pi j\omega \mu_0 f_n} - f_n} \qquad \ldots\ldots 4.19$$

This is an important design equation.

Now that we have characterized a single slot module, we can add many such modules one after the other where the n^{th} module offers a normalized series impedance Z^a_n/Z_0 [see fig. 4.3]

Our idea is to calculate the impedance (Z_1'/Z_0) which is the input impedance of the array. For that, we will have to calculate Z_n'/Z_0 at each n^{th} terminal. The last impedance in the array is Z^a_N and then after a line length of $S_{N,N'}$ the line is terminated in an impedance Z_T. The separating lengths ($S_{i,i+1}$) from ith to

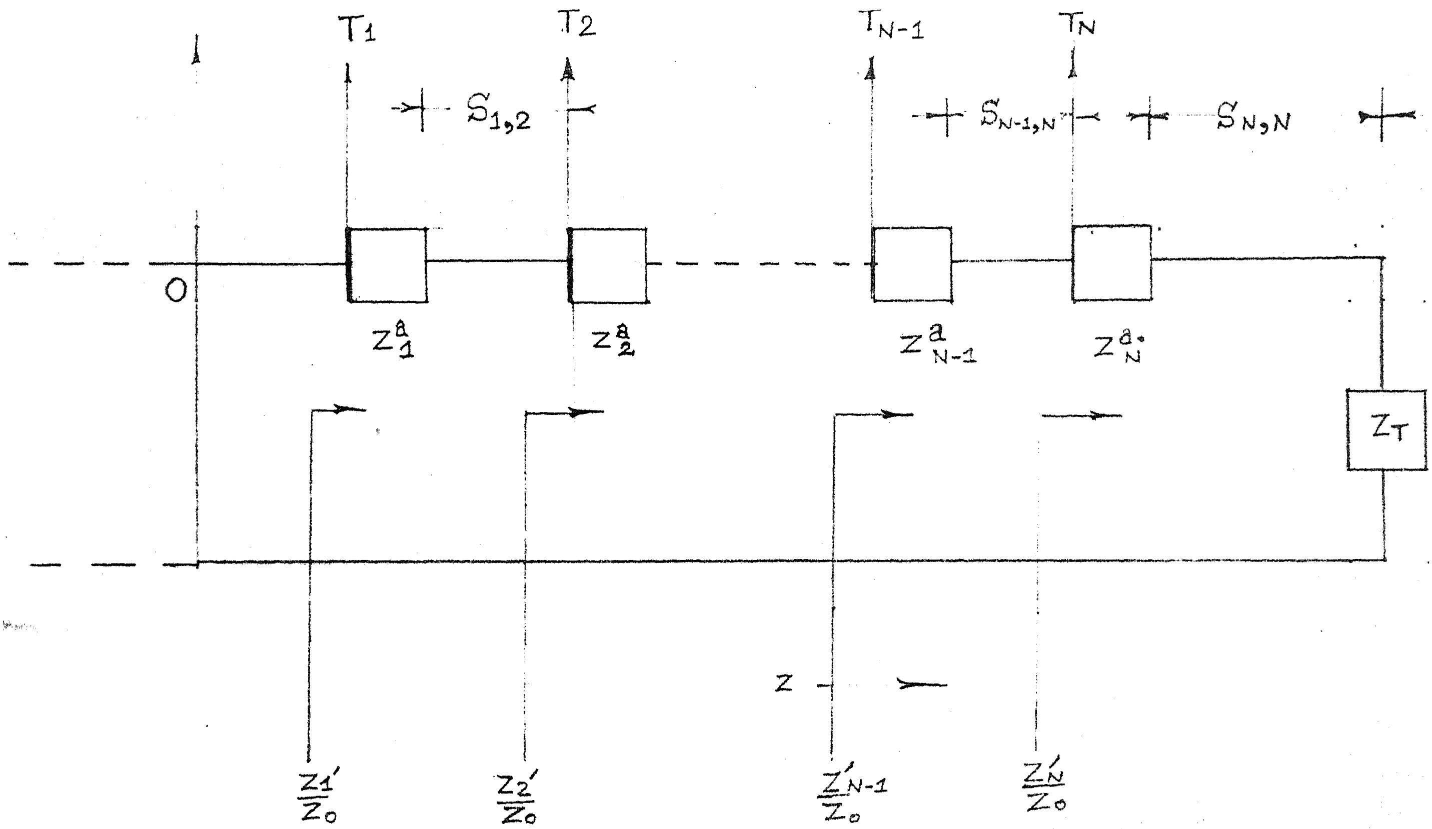


Fig 4.3. Equivalent array of lumped impedances.

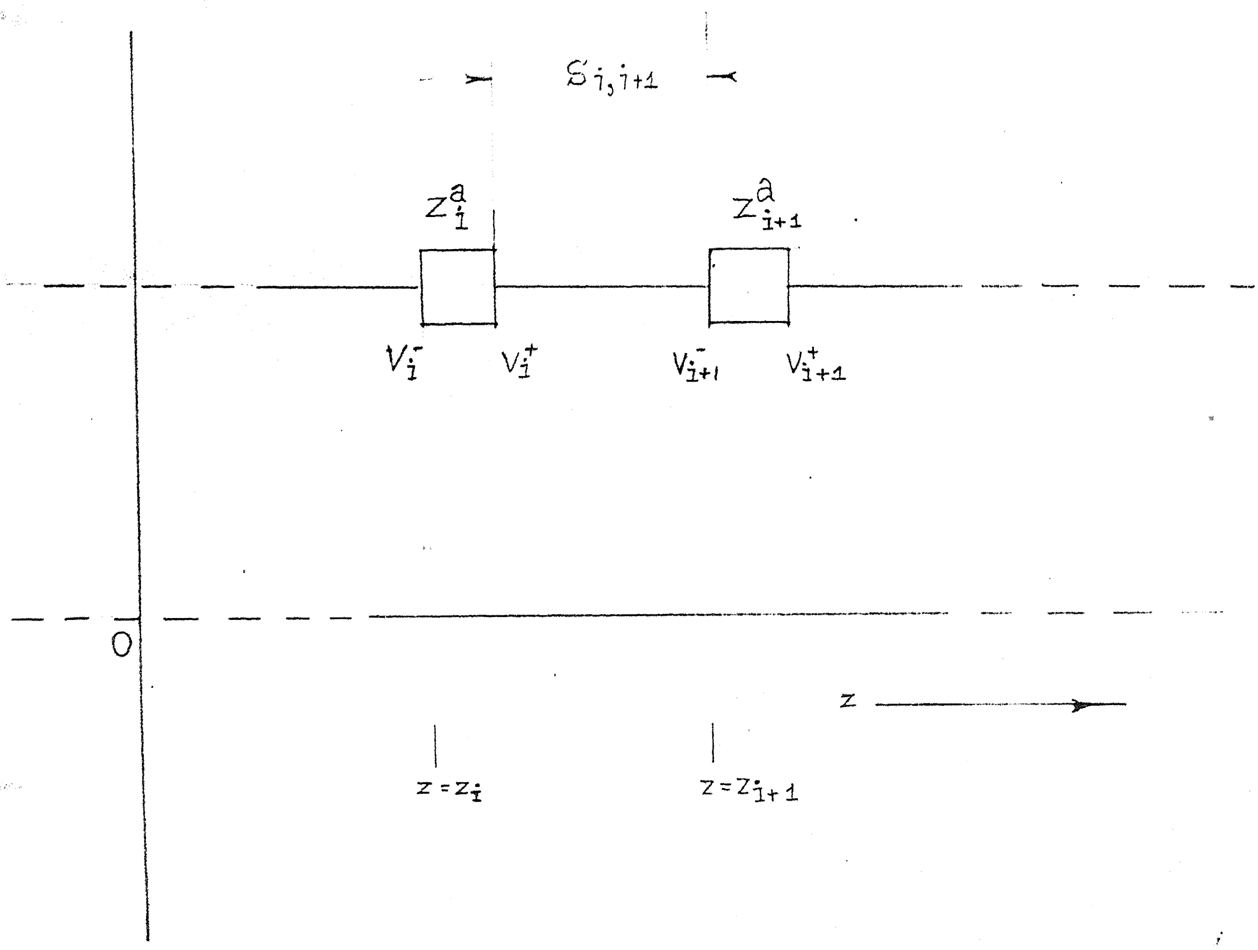


Fig 4.4. For isolated junctions i and $i + 1$.

i+1th module need not be the same as the actual distance between the slots.

The input impedance looking at the Nth terminal TN is

$$\frac{Z'_N}{Z_o} = \frac{Z^a_N}{Z_o} + \frac{\frac{Z_T}{Z_o} + j\tan\beta S_{N,N}}{1 + j\frac{Z_T}{Z_o}\tan\beta S_{N,N}} \quad \ldots\ldots\ldots 4.20$$

The input impedance at the nth terminal T_n is,

$$\frac{Z'_n}{Z_o} = \frac{Z^a_n}{Z_o} + \frac{\frac{Z'_{n+1}}{Z_o} + j\tan\beta S_{n,n+1}}{1 + j\frac{Z'_{n+1}}{Z_o}\tan\beta S_{n,n+1}} \quad \ldots\ldots\ldots 4.21$$

and therefore, at the 1st terminal T_1 is

$$\frac{Z_1}{Z_o} = \frac{Z^a_1}{Z_o} + \frac{\frac{Z'_2}{Z_o} + j\tan\beta S_{1,2}}{1 + j\frac{Z'_2}{Z_o}\tan\beta S_{1,2}} \quad \ldots\ldots\ldots 4.22$$

Since we will also have to know the slot voltages V^s_n we invoke the transmission line equations. Take any junction i, as shown in fig.4.4, the voltage and current are

$$V_i^+ = V_{i+1}^- \cos(K\, s_{i,i+1}) + j\, I_{i+1}^- Z_o \sin(K\, s_{i,i+1}) \quad \ldots\ldots\ldots 4.23$$

$$I_i Z_o = j\, V_{i+1}^- \sin(K\, s_{i,i+1}) + I_{i+1}^- Z_o \cos(K\, s_{i,i+1}) \quad \ldots\ldots\ldots 4.24$$

Where V_i^+ is the voltage, just at the right of the lumped active impedance of the slot and V_i^- is the voltage just at the left of it. Since the slot is a series impedance, the currents I_i^-, I_i^+ are equal and

$$I_i^- = I_i^+ = I_i$$

Hence, $V_i^- = Z^a{}_i\, I_i + V_i^+$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.25$

And $\Delta V_i = V_i^- - V_i^+ = Z^a{}_i\, I_i$ 4.26

Then equations 4.23 and 4.24 becomes

$$(V_i^- - I_i Z_i^a) = V_{i+1}^- \cos(K\, s_{i,i+1}) + j\, I_{i+1} Z_o \sin(K\, s_{i,i+1}) \quad .4.27$$

$$I_i = j\frac{V_{i+1}^-}{Z_o}\sin(K\, s_{i,i+1}) + I_{i+1}\cos(K\, s_{i,i+1}) \quad \ldots\ldots\ldots\ldots 4.28$$

We have replaced I_{i+1}^- by I_{i+1}. Writing them in a matrix form

$$\begin{bmatrix} V_i^- - I_i Z_i^a \\ I_i \end{bmatrix} = \begin{bmatrix} \cos(K\, s_{i,i+1}) & jZ_o \sin(K\, s_{i,i+1}) \\ j\frac{1}{Z_o}\sin K\, s_{i,i+1} & \cos K\, s_{i,i+1} \end{bmatrix} \begin{bmatrix} V_{i+1}^- \\ I_{i+1} \end{bmatrix} \quad \ldots\ldots 4.29$$

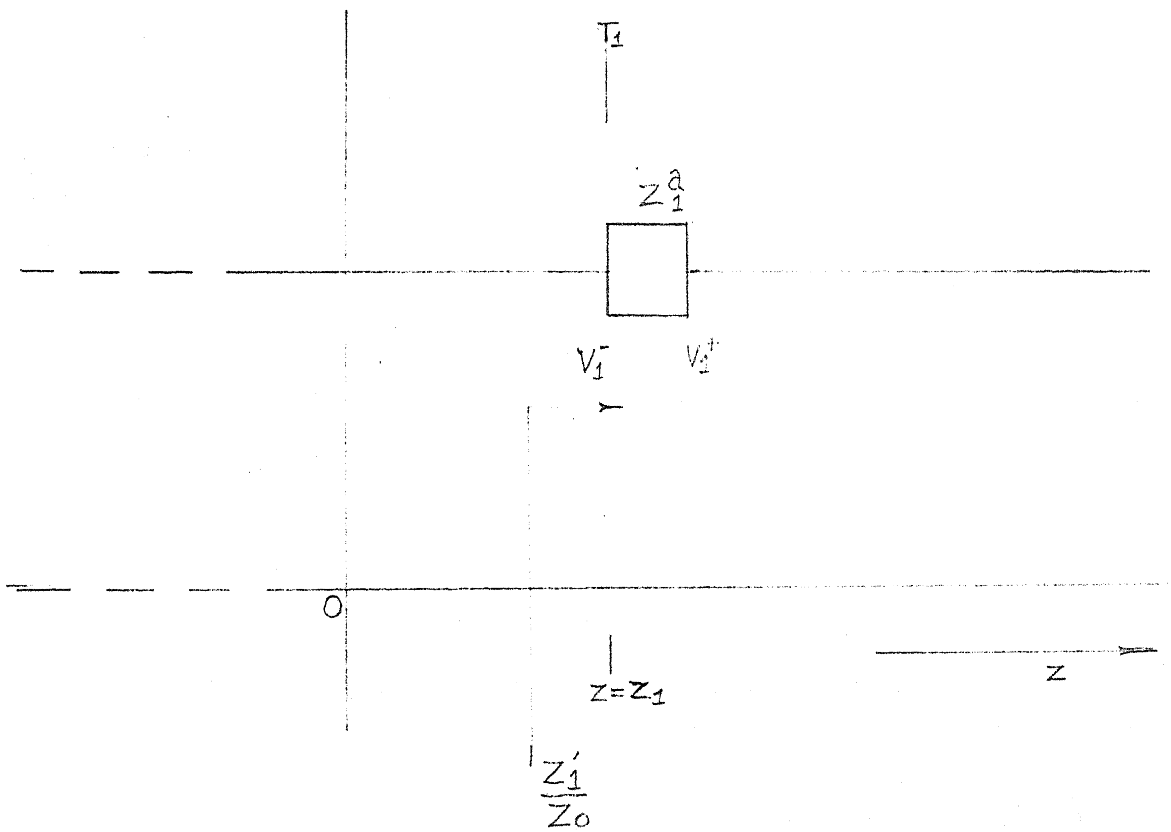


Fig 4.5. First module.

which can be written in a compact form as ,

$$\begin{bmatrix} V_i^- - I_i Z^a_i \\ I_i \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} V_{i+1}^- \\ I_{i+1} \end{bmatrix} \qquad \ldots\ldots\ldots 4.30$$

where,

$$\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} \cos K\, s_{i,i+1} & jZ_o \sin K\, s_{i,i+1} \\ j \dfrac{\sin}{Z_o} K\, s_{i,i+1} & \cos K\, s_{i,i+1} \end{bmatrix} \qquad \ldots\ldots\ldots 4.31$$

Therefore, from 4.30, we obtain,

$$\begin{bmatrix} V_{i+1}^- \\ I_{i+1} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}^{-1} \begin{bmatrix} V_i^- - I_i Z_i^a \\ I_i \end{bmatrix} \qquad \ldots\ldots\ldots 4.32$$

Consider the 1st terminal T_1 as shown in fig. 4.5. The voltage and current at the left of T_1 are given by (for $z<z_1$),

$$V_1^- = e^{-j\beta(z-z_1)} + \Gamma\, e^{+j\beta(z-z_1)} \qquad \ldots\ldots\ldots 4.33$$

$$I_1 = \frac{1}{Z_o} e^{-j\beta(z-z_1)} - \Gamma\, e^{+j\beta(z-z_1)} \cdot \frac{1}{Z_o} \qquad \ldots\ldots\ldots 4.34$$

where it is assumed that a wave of unit amplitude is incident from left and some part of it gets transmitted feeding the slots and some gets reflected. The reflection co-efficient is

$$\Gamma = \left(\frac{Z_1'}{Z_o} - 1 \right) \Big/ \left(\frac{Z_1'}{Z_o} + 1 \right) \qquad \ldots\ldots\ldots 4.35$$

as the input impedance at terminal T_1 is Z_1' / Z_o. The current and the voltage at the terminal 1 are

$$I_1 = \frac{1-\Gamma}{Z_o} = \frac{2}{Z_1' + Z_o} \qquad \ldots\ldots 4.36$$

$$V_1^- = 1 + \Gamma = \frac{2 Z_1'}{Z_1' + Z_o} \qquad \ldots\ldots 4.37$$

combining 4.16 and 4.25 we get,

$$V^s_1 = \frac{I_1 Z^a_1}{2 f_1 \sqrt{Z_o}} \qquad \ldots\ldots 4.38$$

substituting 4.38 and 4.36 in 4.32, V_2^s and I_2 are calculated.

so

$$V^s_2 = \frac{I_2 Z^a_2}{2 f_2 \sqrt{Z_o}} \qquad \ldots\ldots 4.39$$

Thus continuing in this fashion, we evaluate slot voltages,

$$V^s_i = \frac{I_i Z^a_i}{2 f_i \sqrt{Z_o}} \qquad \ldots\ldots 4.40$$

The calculation of effective slot impedance follows an iterative procedure as explained below :

Equation 4.19 gives the expression for effective impedance of the slot. It is seen that to determine Z^{a}_{n}/Zo, one has to know the distribution of slot voltages, which are once again related to Z^{a}_{n}/Zo via 4.40. Hence an iterative procedure has been used to evaluate V^{s}_{m}/V^{s}_{n} or Z^{a}_{n}/Zo.

To start the iteration, first the mutual impedances among the slots are neglected and all Z^{a}_{n}/Zo turn out of be Z_{n}/Z_{o} which is the self simpedance of the n^{th} slot. Equation 4.36 and 4.37 are evaluated next. From 4.38 V^{s}_{1} is determined. Evaluating 4.32, V_{2}^{-} and I_{2} are found out next and consequently V^{s}_{2}. Following the same procedure V^{s}_{n} is evaluated for any n. Now V^{s}_{n} are known (initial values), for all n, Z^{a}_{r}/Zo can be calculated by putting these V^{s}_{m} and V^{s}_{n} in 4.19. Knowing all the Z^{a}_{n}/Zo we find Z_{1}/Z_{0}. And then by the same procedure as written above V^{s}_{n} are known in this second iteration. And so, the iteration continues. It is observed that very few iterations, 2 or 3 are sufficient.

CHAPTER 5

DESIGN OF LPASS

5.1. INTRODUCTION :

This chapter explains what is meant by log periodic array. The different variables associated with LPASS, for example, lengths of the slots, widths of the slots, spacings between the slots etc., are evaluated and the method of evaluation is discussed in detail. The parameter sigma (σ) is introduced which is one fourth of the ratio of spacing between the first and second slot of an LPASS and the length of the second slot.

Given f_1 and f_2 the operating bandwidth required, in the symmetric distribution, the first and the last slot resonant frequencies are chosen symmetrically about the centre frequency $(f_1+f_2)/2$. In the asymmetric distribution, equal number of slots are placed on either side of the centre frequency, with one slot resonant at the centre frequency. This makes the number of slots n, odd. For n even, n-1 slots are distributed in the previous manner, and the n^{th} slot is placed at the low frequency end. Whether n is odd or even, the distribution of the first and the last resonant frequencies of the slots are asymmetric about the centre frequency, and hence the name asymmetric.

In the end a programme is written to evaluate the parameter values and the algorithm is discussed in detail. Different graphs of VSWR over the bandwidth varying each parameter are also shown and compared.

5.2. DESIGN PROCEDURE :

In chapter 1, we briefly discussed what is a log periodic array and in this chapter we first explain why it is called log periodic.

Let us choose discrete samples of frequency like f_1, f_2, f_3, ... such that they form a Geometric Progression (G.P.) with a common ratio τ,

$$f_2 = f_1 \tau \qquad \ldots\ldots\ldots\ldots\ldots 5.1$$
$$f_3 = f_2 \tau = f_1 \tau^2 \qquad \ldots\ldots\ldots 5.2$$
$$\cdot$$
$$\cdot$$
$$f_n = f1 \, \tau^{n-1} \qquad \ldots\ldots\ldots\ldots\ldots 5.3$$

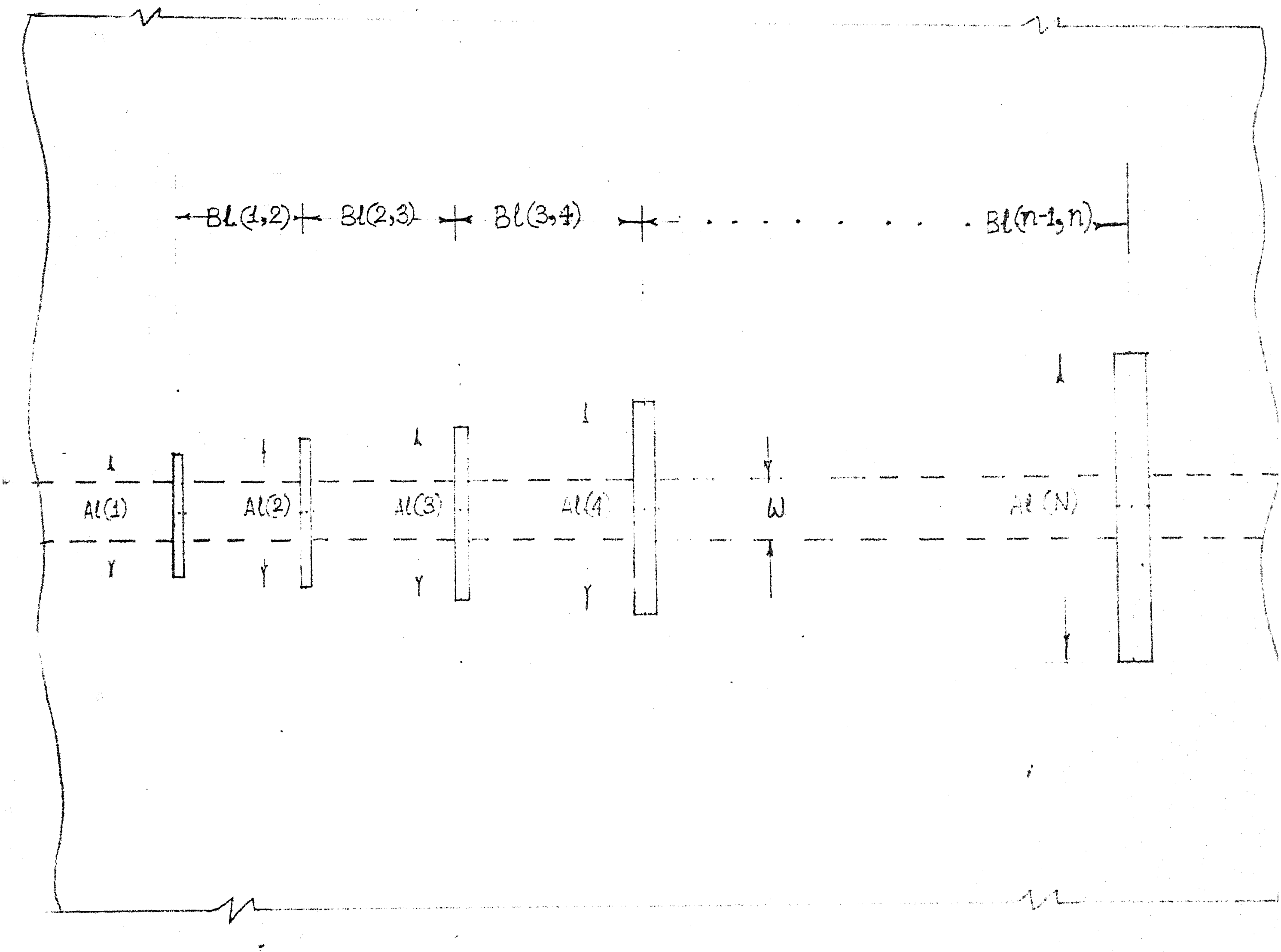


Fig 5.1. A single unit LPASS.

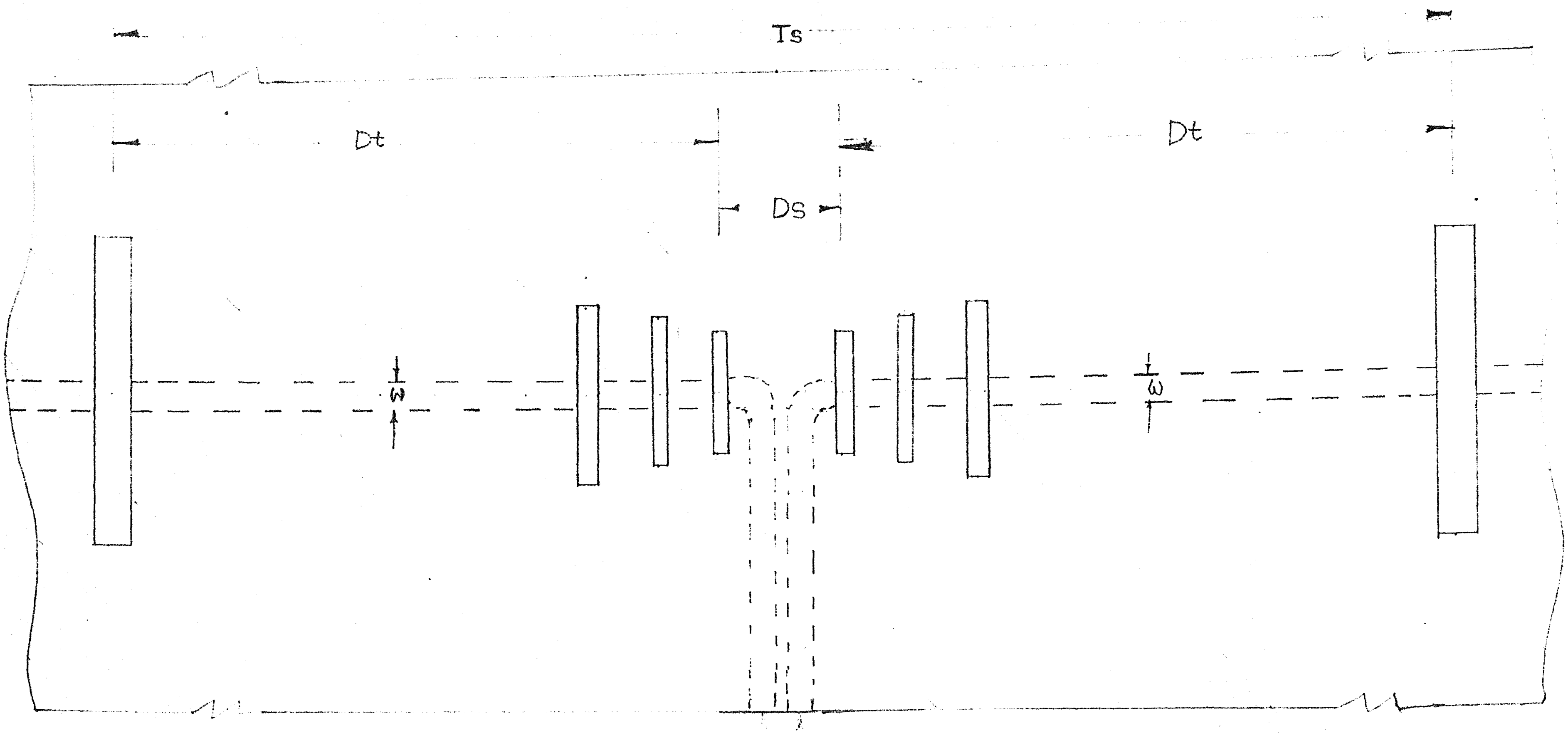


Fig 5.2. Two LPASS units separated by a distance Ds.

Chose a sequence of dipoles or slots D_1, D_2, D_3,....... such that D_1 is resonant at f_1, D_2 at f_2 and so on. Since the resonance shifts from dipole to dipole or slot to slot but the antenna characteristics i.e., input VSWR and pattern remain same at these frequencies, we can say that the characteristics repeat at these frequencies. In otherwords,

$$f_i = f_1 \tau^{i-1} \quad .... 5.4$$

or $$\ln f_i = \ln f_1 + (i-1) \ln \tau ...5.5$$

after a period $\ln\tau$ the antenna characteristics repeat.

In fig. 5.1 a single unit LPASS is shown. Fig. 5.2 shows two units of LPASS on the same upper ground plane of a stripline. The stripline dimensions are shown in table 5.1. They are so chosen as to have the characteristic impedance of the line equal to 50 ohms.

Let the lengths, widths and spacings between slots be denoted as follows:

Al(i) is the length of i^{th} slot,

Bl(i,i+1) is the spacing between i^{th} to $i+1^{th}$ slot,

Dn(i) is the width of the i^{th} slot.

N is the number of slots in a single unit LPASS.

From the principle of operation of LPASS as described in the introduction,

$$Al(2) = Al(1)\tau \quad .... 5.6$$

$$Al(3) = Al(2).\tau = Al(1)\tau^2 ...5.7$$

.

.

$$Al(i) = Al(1)\tau^{i-1} .... 5.8$$

where τ is the common ration of the G.P. sequence and is grater than 1.

Also, $$Bl(i,i+1) = Bl(1,2)\tau^{i-1} ....5.9$$

$$Dn(i) = Dn(1)\tau^{i-1} ....5.10$$

Let us define a quantity called sigma .

$$\sigma = \frac{Bl(1,2)}{4\ Al(2)} \quad ....5.12$$

If sigma and Al(2) are specified Bl(i,i+1) spacings are determind by 5.10.

To find out the distance between two units of LPASS Ds, as shown in fig. 5.2, one has to know the l_{res}/λ_{res} [see chapter 2] of the slots. From chapter

2 it is found to be 0.405336.

Hence $$Ds = \frac{Al(1)}{2 \times 0.405336} \quad \ldots\ldots\ldots.5.12$$

The total spacing required for a single unit LPASS is :

$$Dt = \sum_{i=1}^{n-1} Bl(i,i+1) \quad \ldots\ldots.5.13$$

But the total spacing for a double unit LPASS, as in fig. 5.2 is

$$Ts = Ds + 2.Dt \quad \ldots\ldots\ldots\ldots.5.14$$

The slots can be selected about the centre frequency fo of the desired handwidth in two different ways; Symmetric and Asymmetric.

Symmetric Design of Slots : In this kind of design two resonant frequencies fa and fb are chosen, equally displaced from fo, where fa is the resonant frequency of the first slot and fb is the resonant frequency of the last slot (N^{th}).

From equation 5.4 τ^{n-1} fa = fb

or $$\ln \tau = \frac{\ln(fb/fa)}{n - 1} \quad \ldots\ldots\ldots\ldots.5.15$$

or $$\tau = \exp\left(\frac{\ln(fb/fa)}{n - 1}\right) \quad \ldots\ldots\ldots.5.16$$

Once τ is known, intermediate resonant frequencies f_2, f_3, ...f_{n-1} are determind by equation 5.4. The length of the first slot is determind by

$$Al(1) = 0.405336.c/fb \text{ cm.} \quad \ldots.5.17$$

where C is the velocity of light. Using equations 5.16 and 5.8 all the other lengths of the slots are calculated.

Asymmetric Design : In this kind of design equal number of slots are placed about the centre frequency for n is odd, where n is the number of slots. For n even, n-1 slots are placed as in the previous manner and the N^{th} slot is placed at the low frequency end.

$$Al(1) = (0.405336.c/fo)/\tau^{NS} \quad \ldots..5.18$$

$$N_s = INT(\ (N-1)/2\ ) \quad \ldots\ldots.5.19$$

all other lengths are determined by the procedure described in the previous paragraph.

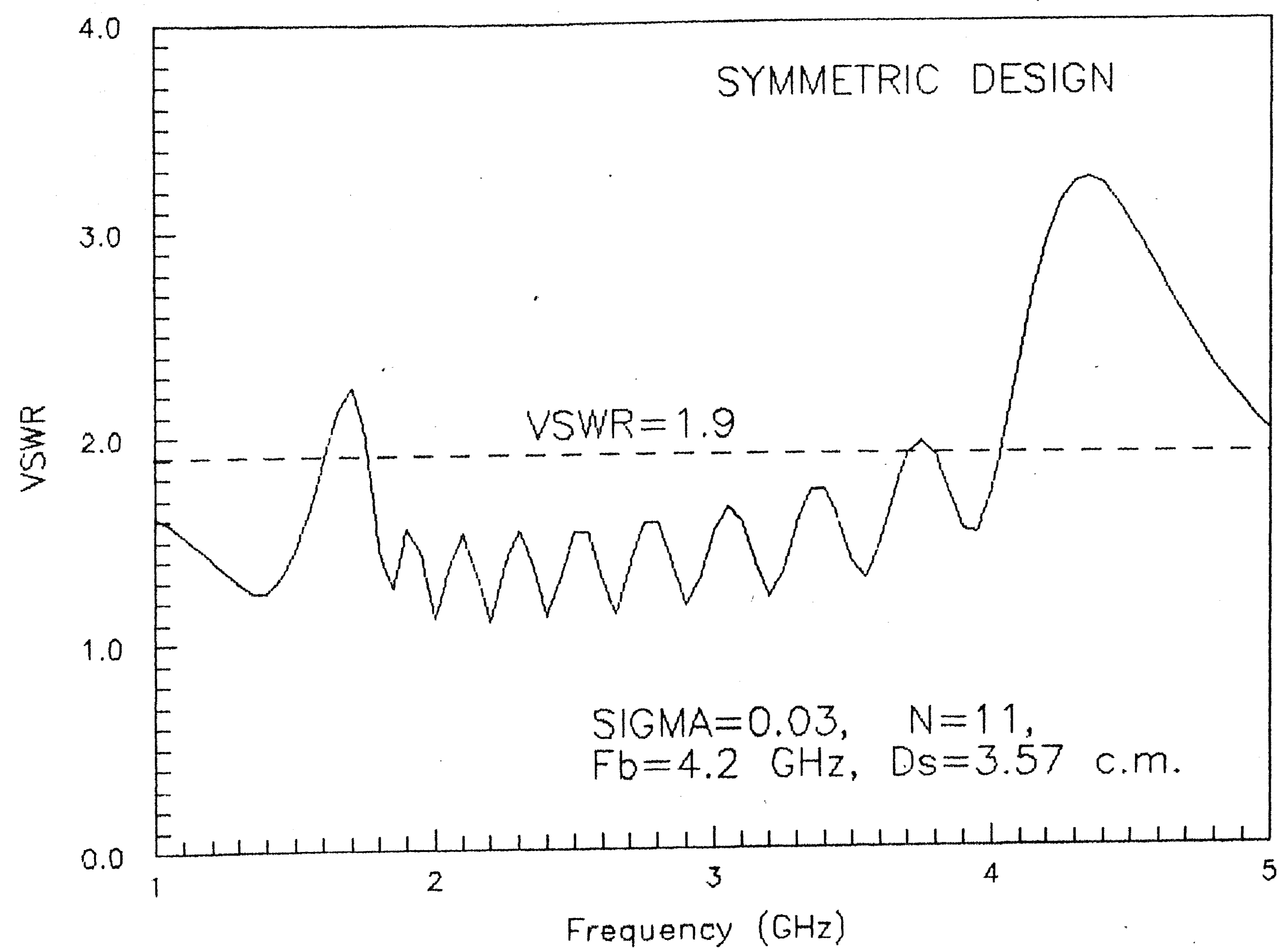


Fig 5.3. VSWR versus Frequency plot for symmetric **design** of LPASS with VSWQ = 1.9.

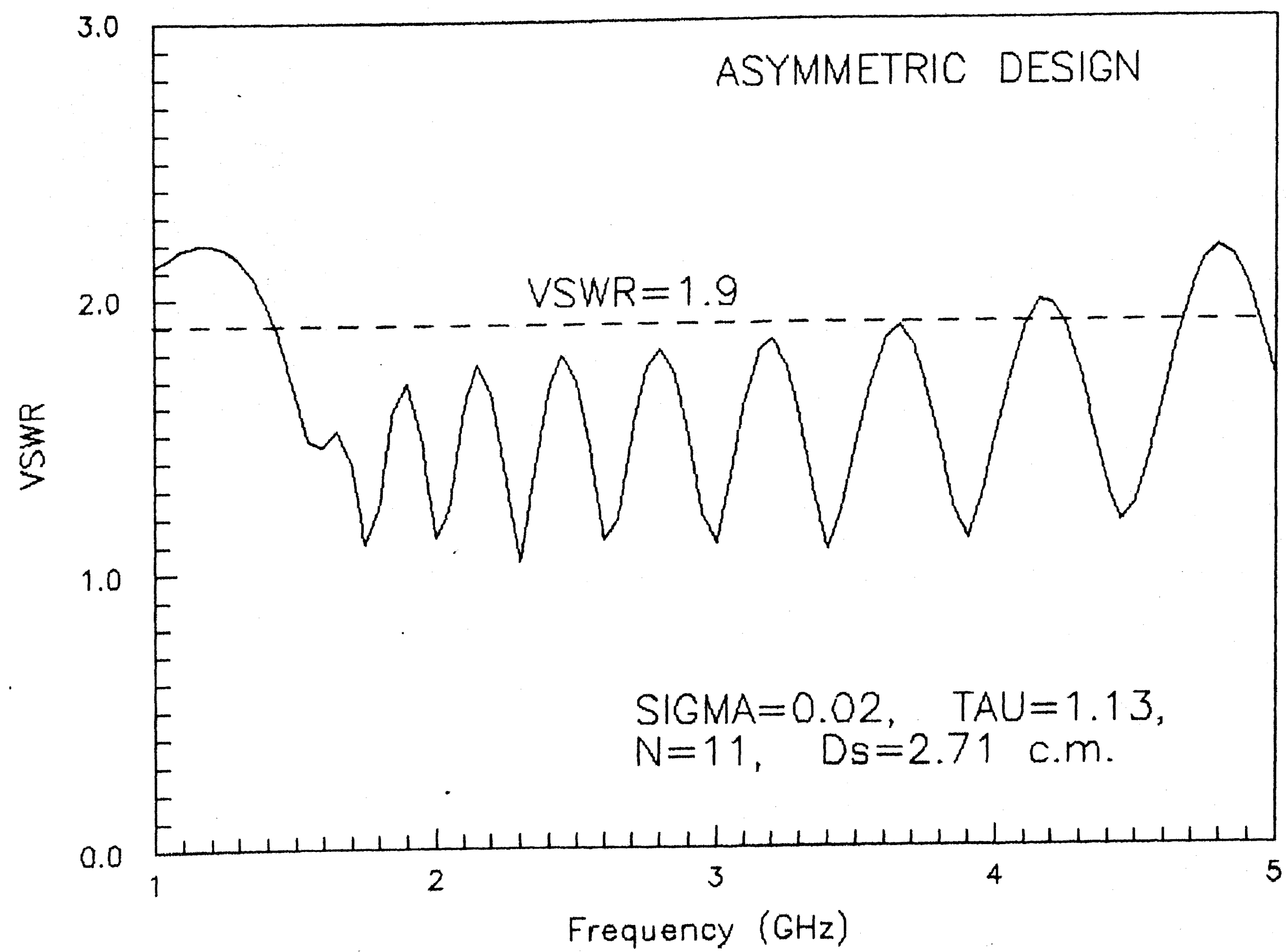


Fig 5.4. VSWR versus Frequency plot for asymmetric design of LPASS with VSWR = 1.9.

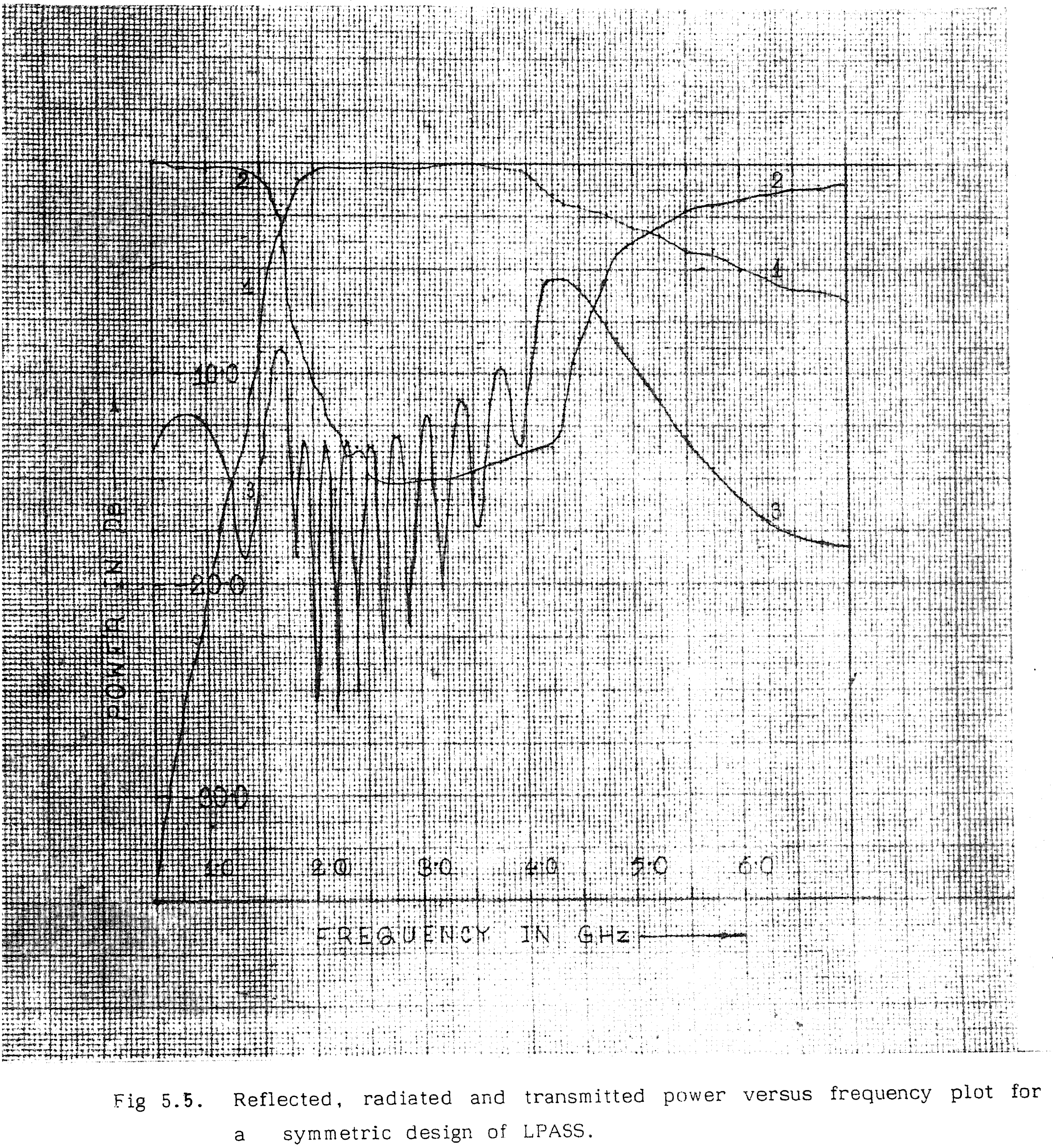


Fig 5.5. Reflected, radiated and transmitted power versus frequency plot for a symmetric design of LPASS.

1. Power radiated by the antenna.
2. Power absorbed by the matched termination.
3. Power reflected at the input.

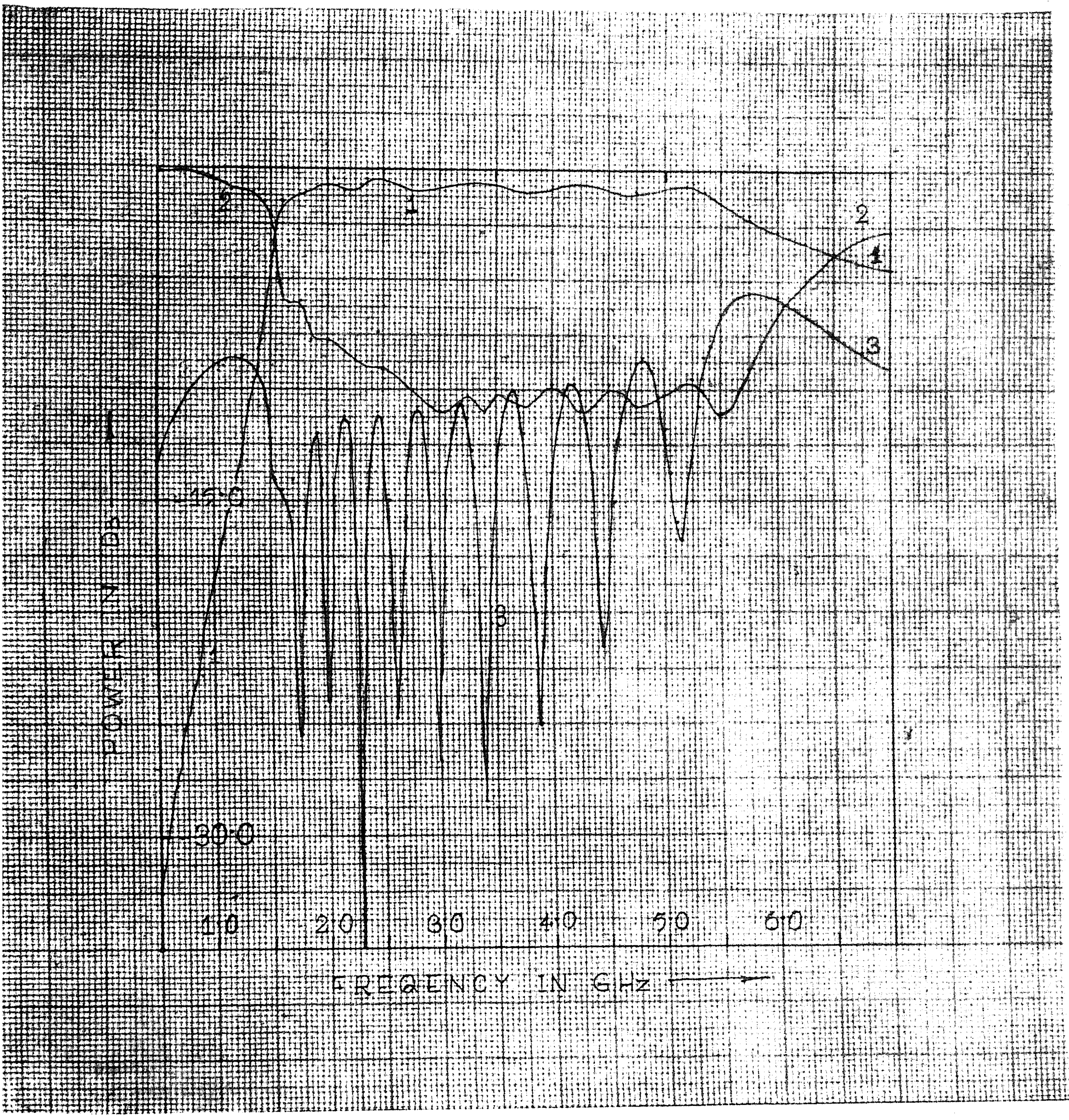


Fig 5.6. Reflected radiated and transmitted power versus frequency plot for an asymmetric design of LPASS.

1. Power radiated by the antenna
2. Power absorbed by the matched termination.
3. Power reflected at the input.

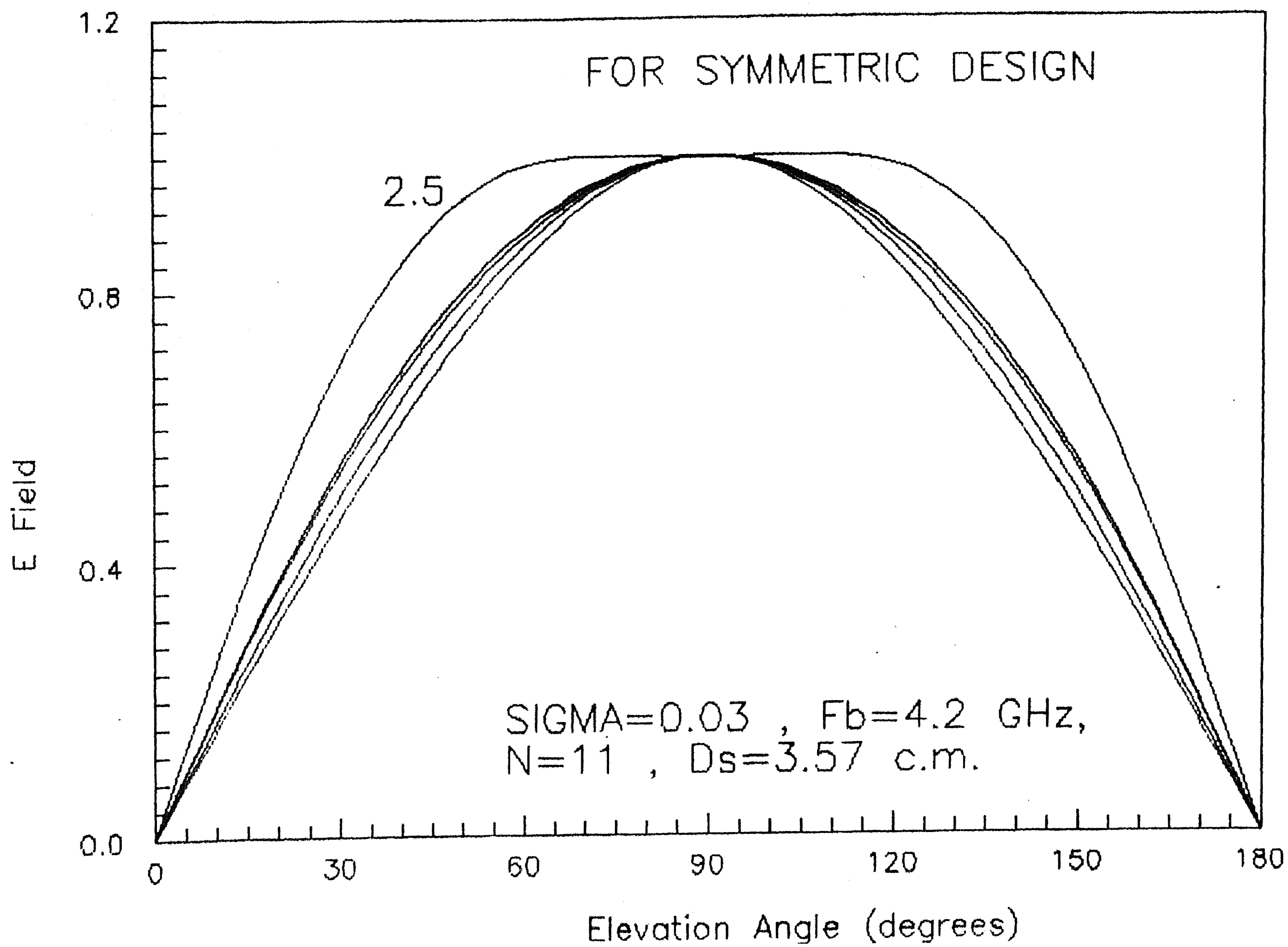


Fig 5.7. Power pattern for a symmetric design for frequencies 2, 2.5, 3.0, 3.5 and 4 GHz.

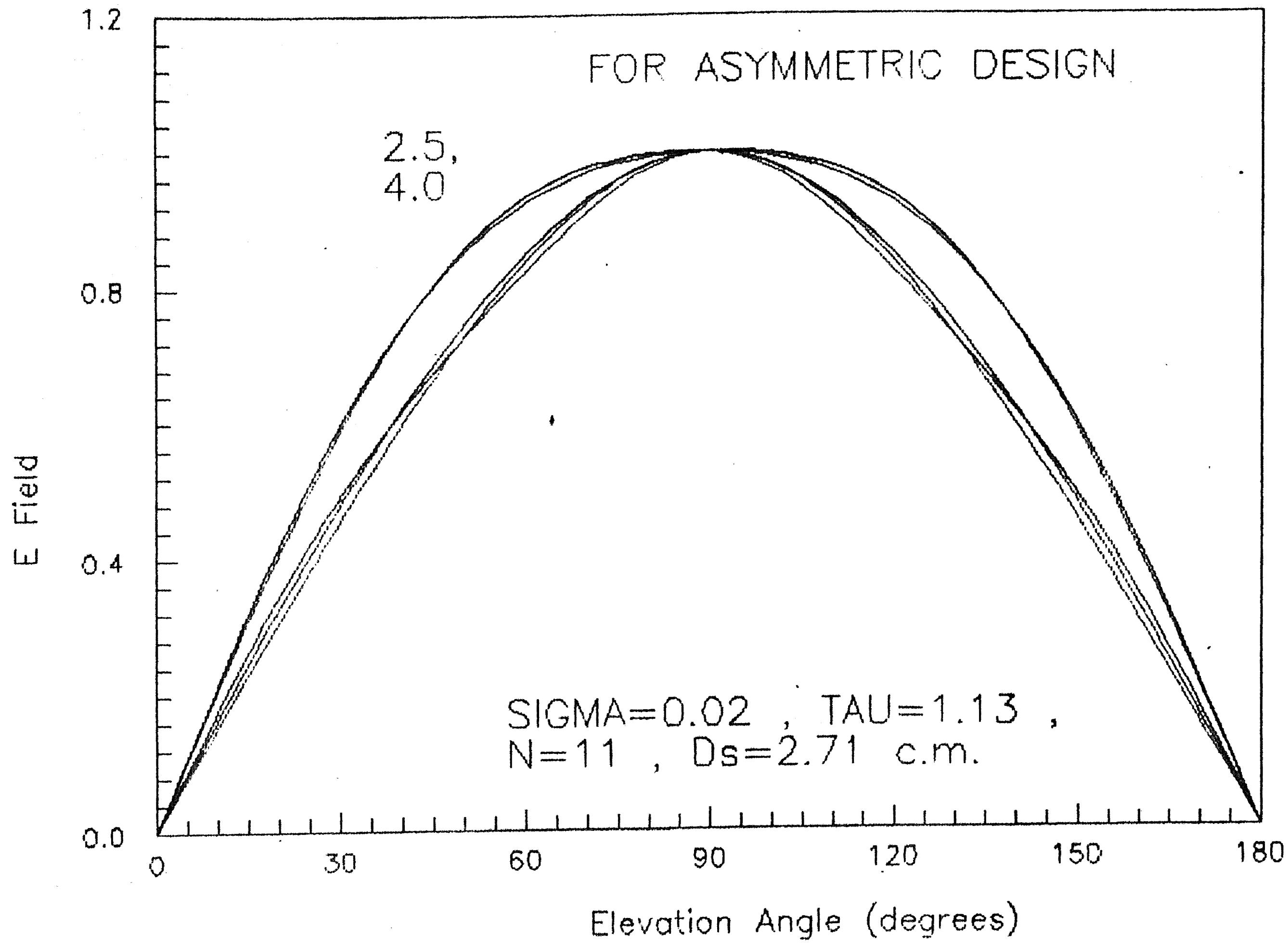


Fig 5.8. Power pattern for asymmetric design for frequencies 2, 2.5, 3.0, 3.5 and 4 GHz.

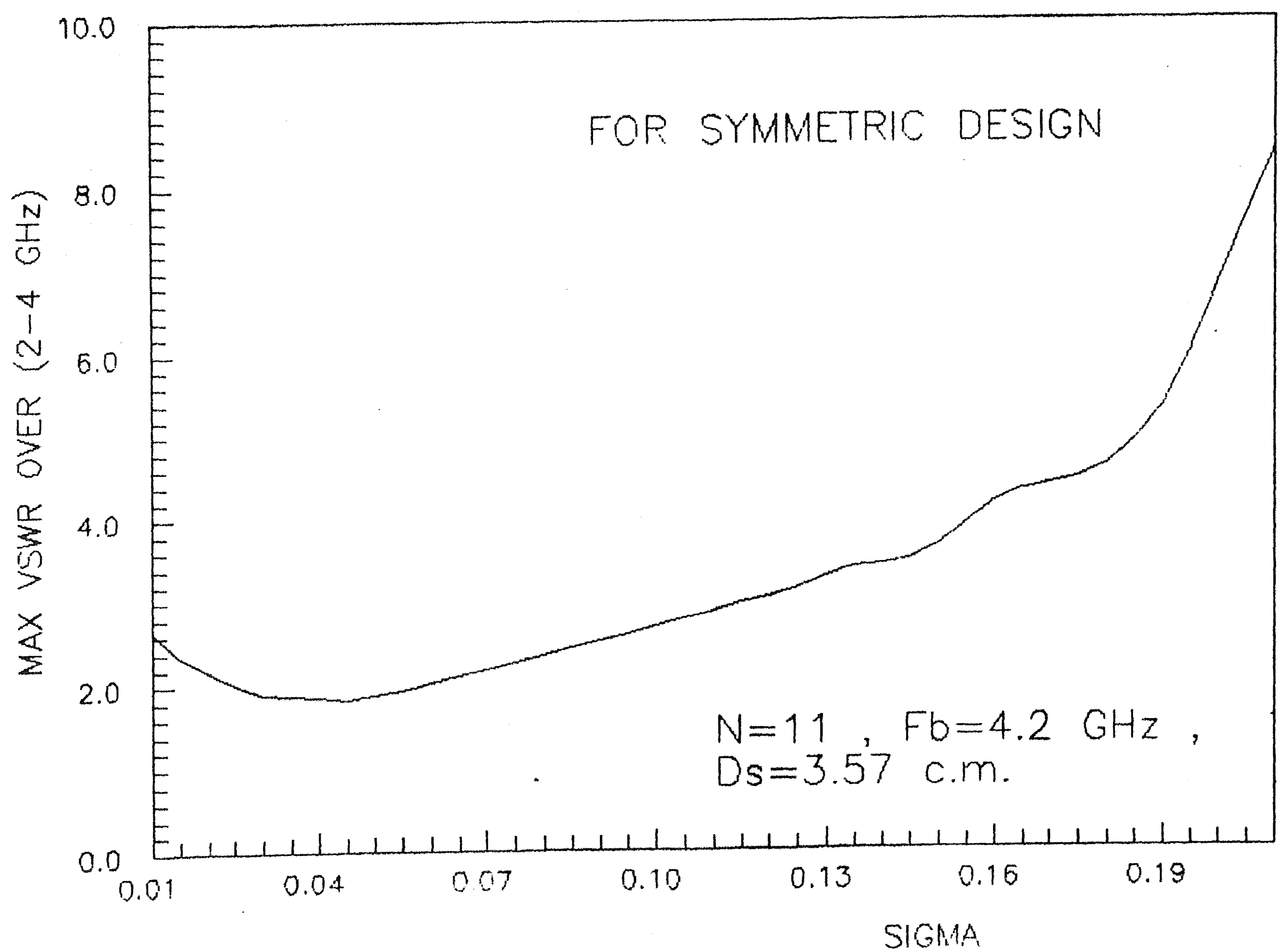


Fig 5.9. Maximum VSWR over 2 - 4 GHz range for variations of sigma.

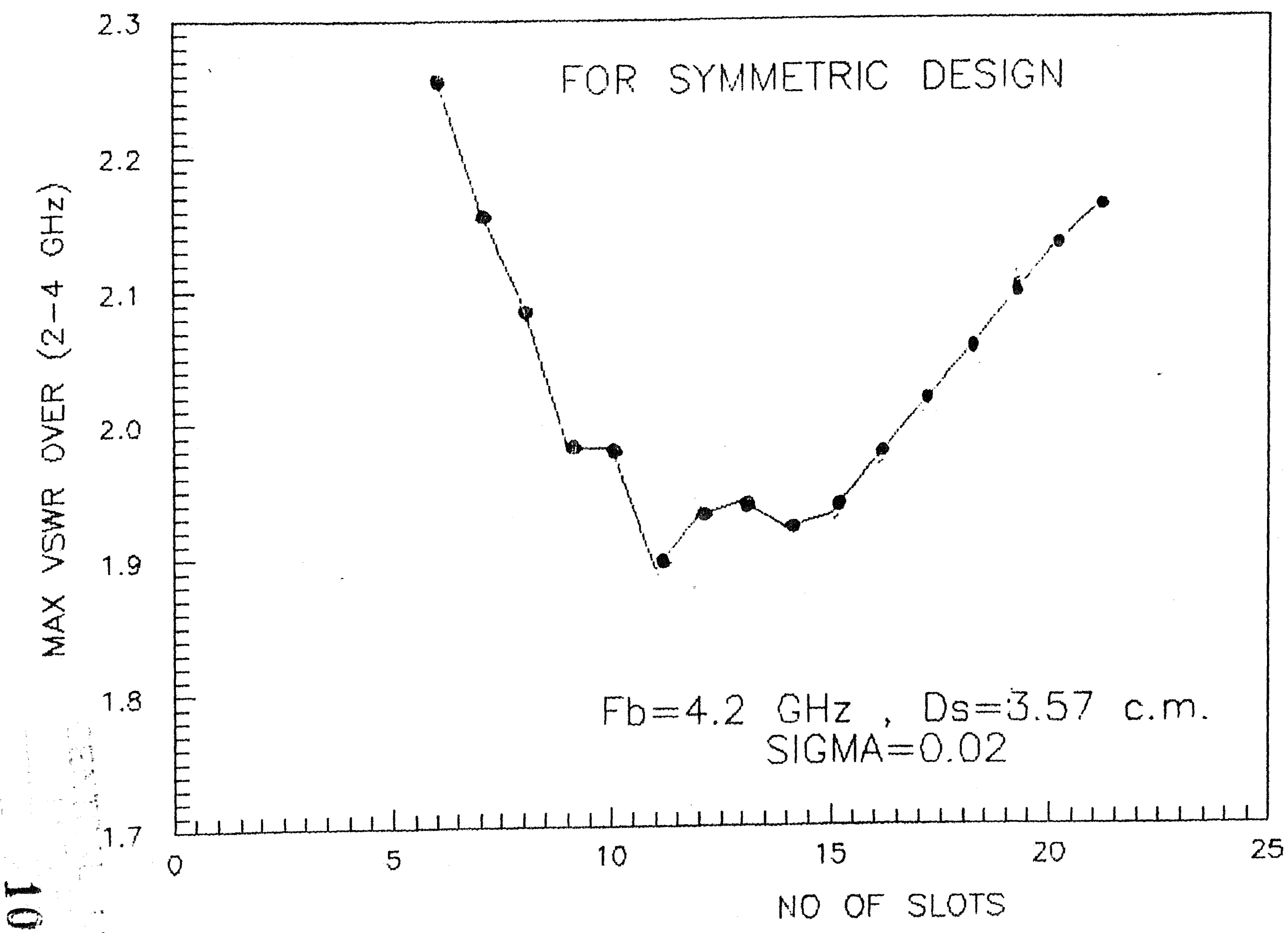


Fig 5.10. Maximum VSWR over 2 - 4 GHz range for variations of number of slots.

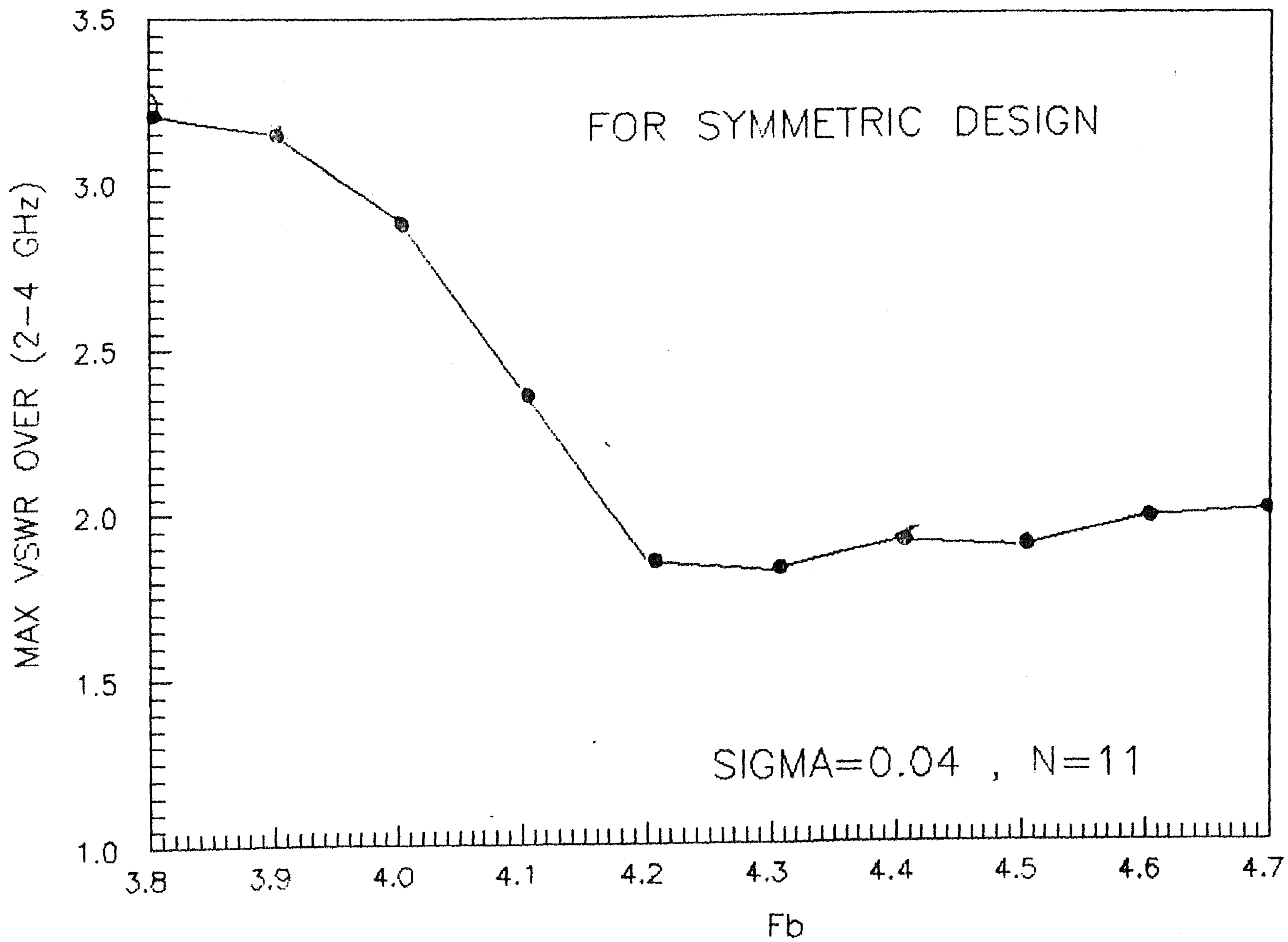


Fig 5.11. Maximum VSWR over 2 - 4 GHz range for variations of fb.

A programme is written to evaluate the equivalent input VSWR of a single unit with the two LPASS units placed side by side as shown in fig. 5.2. First, the mutual coupling is neglected and the maximum VSWR over a bandwidth (2-4GHz) is calculated for various values of σ , N and Fb or Fa. τ is a dependent variable in the symmetric case but in the asymmetric case, it is an independent parameter. In the same programme Ds is also computed. For a particular sets of values of parameters i.e., σ , N, τ etc. maximum VSWR is less than or equal to some number, VSWQ, which is a selection point. Sets of parameter values above this point are neglected.

The total spacing Ts for each selected set is compared and the set which gives minimum Ts, is selected. A value of VSWQ = 1.9 is taken as the cut off point for defining input bandwidth. In this case, the VSWR versus frequency plot is generated taking mutual coupling between slots into account. They are shown in fig. 5.3 and 5.4. Both the Symmetric and the Asymmetric cases are considered. Also the reflected power, power transmitted to the matched load and power radiated by the antenna are plotted versus frequency in the range 0.5 - 7GHz which are shown in fig. 5.5 and 5.6.

5.3. DISCUSSION :

From the VSWR versus σ plot it is seen that for σ very small (close to zero). The maximum VSWR over the operating bandwidth is very high. As σ is increased maximum VSWR decreases upto a certain value and if σ is increased further once again maximum VSWR increases. It is clear that, during computation of optimum set of parameters, if the VSWR computed for three or four consecutive values of σ , and we find that the maximum VSWR is increasing monotonically, further computation in seeking input match by increasing sigma, is unnecessary.

With an increasing N, the maximum VSWR decreases but, here also if N is increased monotonically, maximum VSWR reaches a minimum point and then once again increases. There may be oscillations for a few consecutive values of N around the minimum point. By increasing fb the maximum VSWR decreases but at the cost of extra slots and spacing.

From the VSWR versus frequency graphs it is seen that beyond the operating bandwidth, VSWR may be between 1 - 2 but then it does not necessarily mean that all the input power is radiated by the antenna. Since there is a matched load at low frequency end of the array, power gets transmitted to this load also. To ascertain clearly, whether the antenna is still radiating efficiently, one should

see the figs 5.5 and 5.6 for power radiated by the antenna in the frequency range specified therein. From the consideration of power radiated by the antenna as well as the input VSWR it is observed that for the same operating bandwidth, the asymmetric design gives extra bandwidth and its total area requirement is also small compared to the Symmetric design.

The pattern is seen to be fairly constant over the frequency range as shown in figs. 5.7 and 5.8.

TABLE 5.1

h_1	h_2	w	a
cm.	cm.	cm.	cm.
0.15875	0.31750	0.3067	10.0

CHAPTER 6

SUMMARY AND CONCLUSION

6.1. DISCUSSION ON THE DESIGN OF LPASS :

From 2nd to 5th chapter, we discussed in detail about a single slot analysis, calculation of mutual coupling between slots, a general array of slots and a log periodic array of slots. In these chapters following interesting points are noted :

1. In a single slot analysis, the resonant length of the slot is not at $0.5\lambda_{res}$ but at $0.405336\,\lambda_{res}$ and the normalized resonant resistance decreases monotonically as h1/b is decreased. Therefore, a transmission line, with a given characteristic impedance Zo, can be matched to an array of slots by choosing suitable h1/b.

2. In VSWR versus frequency plots (chapter 5), it is observed that the VSWR alone cannot be the design criterion. The power radiated by the antenna as well as the power transmitted to the matched load, at the end of the array should be taken into account. For efficient radiation, the power input to the antenna should be radiated and not dissipated in the matched termination.

3. For an asymmetric antenna design, with f_1 and f_2 as the two frequencies used in the design, we obtain a bandwidth more than $(f_2 - f_1)$. For a given bandwidth, the asymmetric design results in a smaller substrate area than a symmetric design.

4. In the VSWR versus power plot, once again, although the input VSWR is low beyond the operating bandwidth, the power radiated by the antenna is small and most of the input power is dissipated at the matched termination.

6.2. FUTURE PROSPECT OF WORK :

The present thesis may be taken as a groundwork of log periodic slot array. For an optimum design of a wideband antenna whose pattern remains the same in the operating bandwidth, one can optimize the number of slots, substrate area etc.. To do this, one may manipulate asymmetric design by choosing a suitable centre frequency which is not the centre frequency of the operating bandwidth. Around the chosen centre frequency, the slots may be placed as per asymmetric design procedure to give the best performance. Also, the slots in both

the symmetric as well as the asymmetric cases may be perturbed from their log periodic position, creating a new arrangment, no longer logperiodic, but gives an optimum performance in terms of radiated power as well as input VSWR.

By listing the values of maximum VSWR over various bandwidths with variations of σ, N, τ, an empirical design formula for the number of slots required, relating σ and τ may be found out.

In the present analysis and design we have assumed pin curtains around each slot but, fabricationwise it might turn out to be difficult. To get rid of pins, to simplify the fabrication and reduce the cost, the analysis of a single slot will have to take into account all higher order modes, propagating as well as nonpropagating. The behaviour of slot impedance with frequency will change. In the analysis of array, the propagating modes have to be considered separately. Looking at the advantage of elimination of pin curtains, this idea appears to be attractive, but the analysis is more involved.

THE END

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